Nonrelativistic String Theory and T-Duality

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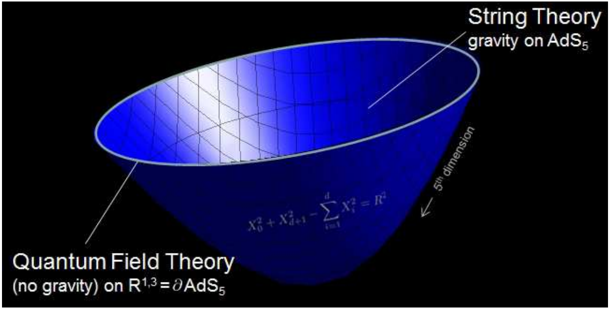
Motivation

Holography



Gravity is not only used to describe the gravitational force!

Non-relativistic Holography

two approaches

- Keep general relativity in the bulk but take background geometry with non-relativistic isometries

  Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

- Take non-relativistic gravity in the bulk

  Gomis, Ooguri (2001); Gopakumar, Bagchi (2009)

Outline

String Newton-Cartan Gravity

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String Newton-Cartan Gravity

Non-relativistic String Theory

T-duality

Actions

Comments

Galilei Symmetries

- time translations : $\delta t = \xi^0$ but not $\delta t = \lambda^i x^i$!
- space translations : $\delta x^i = \xi^i \qquad i = 1, 2, 3$
- spatial rotations : $\delta x^i = \lambda^i{}_j \, x^j$
- Galilean boosts : $\delta x^i = \lambda^i t$

Absolute Time

$$\tau_{\mu\nu} \equiv \partial_{[\mu}\tau_{\nu]} = 0 \quad \to \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \quad \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} \mathrm{d}x^\mu \tau_\mu = \int_{\mathcal{C}} \mathrm{d}\rho \quad \text{is path-independent}$$

From Galilei to Bargmann

the zero commutator

$$[G_A, P_B] = 0$$

implies that a massive particle with non-zero spatial momentum P_B cannot by any boost transformation G_A be brought to a rest frame $\Rightarrow$

$$[G_A, P_B] = \delta_{AB} M \quad \rightarrow \quad \text{extra gauge field } m_\mu$$

NC Gravity couples to particles

what about strings?

String Galilei Symmetries

$$D+1 \text{ flat indices} \rightarrow \begin{cases} 2 \text{ longitudinal indices } A \\ D-1 \text{ transverse indices } A' \end{cases}$$

longitudinal translations	H_A
transverse translations	$P_{A'}$
string Galilei boosts	$G_{AB'}$
longitudinal Lorentz rotations	M_{AB}
transverse spatial rotations	$J_{A'B'}$

Geometrical Constraints

$$D_{[\mu}\tau_{\nu]}{}^{A} = 0 \quad \text{with} \quad \tau_{\mu}{}^{A} \ \text{generalized clock function}$$

- conventional constraints
- geometrical constraints

Noncentral Extension

$$[G_{AA'}, P_{B'}] = 0 \qquad \rightarrow \qquad [G_{AA'}, P_{B'}] = \delta_{A'B'} Z_A$$

The independent string NC fields $\{\tau_\mu{}^A, e_\mu{}^{A'}, m_\mu{}^A\}$ transform as follows:

$$\begin{aligned}
\delta\tau_\mu{}^A &= \Lambda^A{}_B \tau_\mu{}^B\,,\\
\delta E_\mu{}^{A'} &= \Lambda^{A'}{}_{B'} E_\mu{}^{B'} - \Sigma_A{}^{A'} \tau_\mu{}^A\,,\\
\delta m_\mu{}^A &= D_\mu \sigma^A + \Sigma^A{}_{A'} E_\mu{}^{A'}
\end{aligned}$$

spatial metric: $H_{\mu\nu} \equiv E_\mu{}^{A'} E_\nu{}^{B'} \delta_{A'B'} + \left(\tau_\mu{}^A m_\nu{}^B + \tau_\nu{}^A m_\mu{}^B\right) \eta_{AB}$

Outline

NR Limit Polyakov Particle

$$S_{\text{Pol.}} = -\tfrac{1}{2}\int d\tau\Big\{-\frac{1}{e}E_\mu{}^A\dot{x}^\mu E_\nu{}^B\dot{x}^\nu\eta_{AB} + M^2 e - 2M M_\mu\dot{x}^\mu\Big\}$$

$$\begin{aligned} S_{\text{Pol.}}(c^2) &= -\tfrac{1}{2}\int d\tau\frac{1}{e}c^2\big[\tau_\mu\dot{x}^\mu - me\big]^2 \\ &= -\tfrac{1}{2}\int d\tau\frac{1}{e}\Big\{\lambda(\tau_\mu\dot{x}^\mu - me) - \frac{1}{4c^2}\lambda^2\Big\} \quad \Rightarrow \end{aligned}$$

$$S_{\text{Pol.}}(\text{N.R.}) = -\tfrac{1}{2}\int d\tau\frac{1}{e}\Big\{\dot{x}^\mu\dot{x}^\nu H_{\mu\nu} + \lambda\big(\tau_\mu\dot{x}^\mu - me\big)\Big\}$$

The Nonrelativistic Polyakov String

$$h_{\alpha\beta} = e_\alpha{}^a e_\beta{}^b \eta_{ab}$$

$$e_\alpha \equiv e_\alpha{}^0 + e_\alpha{}^1 \,, \qquad \bar{e}_\alpha \equiv e_\alpha{}^0 - e_\alpha{}^1$$

$$\tau_\mu \equiv \tau_\mu{}^0 + \tau_\mu{}^1 \,, \qquad \bar{\tau}_\mu \equiv \tau_\mu{}^0 - \tau_\mu{}^1$$

$$S_{\text{Pol.}} = -\frac{T}{2} \int d^2\sigma \left[\sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha x^\mu \partial_\beta x^\nu H_{\mu\nu} + \epsilon^{\alpha\beta} \left(\lambda\, e_\alpha \tau_\mu + \bar{\lambda}\, \bar{e}_\alpha \bar{\tau}_\mu \right) \partial_\beta x^\mu \right]$$

$$- \frac{T}{2} \int d^2\sigma\, \epsilon^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x^\nu B_{\mu\nu}$$

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Relativistic T-duality

Buscher (1987,1988); Roček, Verlinde (1992)

adapted coordinates: $x^\mu = (y, x^i)$ $\qquad k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = \underbrace{S_{\text{Pol.}}(\partial_\alpha y \to v_\alpha)}_{\text{quadratic in } v_\alpha!} - T \int d^2\sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$\frac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \rightarrow \quad v_\alpha$ is solved for in terms of dual coordinate $\tilde{y}$

$$\tilde{G}_{yy} = \frac{1}{G_{yy}} \qquad \Rightarrow \qquad R \Leftrightarrow \frac{1}{R}$$

Non-relativistic Longitudinal T-duality

$$\tau_\mu{}^0 k^\mu = 0, \qquad \tau_\mu{}^1 k^\mu \neq 0\,, \qquad E_\mu{}^{A'} k^\mu = 0$$

adapted coordinates: $x^\mu = (y, x^i) \qquad k^\mu \partial_\mu = \partial_y$

$$S_{\text{parent}} = S_{\text{Pol.}}(\partial_\alpha y \to v_\alpha) - T \int d^2\sigma \epsilon^{\alpha\beta} \tilde{y} \partial_\alpha v_\beta$$

$\dfrac{\delta S_{\text{parent}}}{\delta v_\alpha} = 0 \quad \to \quad v_\alpha$ is solved for (in terms of λ and $\bar{\lambda}$!)

Dual Action

$$\tilde{S}_{\text{long.}} = -\frac{T}{2}\int d^2\sigma \left(\sqrt{-h}\, h^{\alpha\beta}\, \partial_\alpha \tilde{x}^\mu \partial_\beta \tilde{x}^\nu\, \tilde{G}_{\mu\nu} + \epsilon^{\alpha\beta}\, \partial_\alpha \tilde{x}^\mu \partial_\beta \tilde{x}^\nu\, \tilde{B}_{\mu\nu}\right)$$

$$\text{with} \quad \tilde{x}^\mu = (\tilde{y}, x^i) \quad \text{and} \quad \tilde{G}_{yy} = 0: \ \text{lightlike direction}$$

- The longitudinal spatial T-dual of the NR string is the Polyakov string moving in a GR background with a lightlike direction
  - DLCQ relativistic string?

- The transverse spatial T-dual of the NR string is again a NR string with a transverse spatial isometry direction à la Buscher

Buscher (1987,1988); Roček, Verlinde (1992)

Outline

Particles in 3D

$$\hat{S} = \frac{1}{2\kappa^2}\int d^3x \left(\hat{E}\,\hat{R}(\hat{\Omega}) - \epsilon^{\mu\nu\rho}\hat{B}_\mu\partial_\nu\hat{A}_\rho\right) \quad \Rightarrow$$

3D Extended Bargmann Gravity

Rosseel + E.B. (2016); Hartong, Obers (2016)

- The 2 central charges originate form the 2 vector fields
- 3D Extended Bargmann Gravity $\neq$ 3D NC gravity

Strings in 4D

$$\hat{S} = \frac{1}{2\kappa^2} \int d^4x \left(\hat{E}\, \hat{R}(\hat{\Omega}) - \epsilon^{\mu\nu\rho\sigma}\, \hat{B}_{\mu\nu} \partial_\rho \hat{A}_\sigma \right)$$

Grosvenor, Şimşek, Yan + E.B., work in progress

- The 2-form gives rise to the 2 noncentral extensions Z_A $(A = 0, 1)$ of the String Bargmann Algebra
- The 1-form gives rise to the extra central extension S of the Extended String Bargmann Algebra

The 4D Extended String Bargmann Gravity Action

Grosvenor, Şimşek, Yan + E.B. (to appear)

$$S = \frac{1}{2\kappa^2}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}\Big[\tfrac{1}{2}\epsilon_{A'B'}E_\mu{}^{A'}E_\nu{}^{B'}\mathcal{R}_{\rho\sigma}(M) - \epsilon_{AB}\epsilon_{A'B'}\tau_\mu{}^{A}E_\nu{}^{A'}\mathcal{R}_{\rho\sigma}{}^{BB'}(G)$$

$$+\,\epsilon_{AB}\tau_\mu{}^{A}m_\nu{}^{B}\mathcal{R}_{\rho\sigma}(J) + \tfrac{1}{2}\epsilon_{AB}\tau_\mu{}^{A}\tau_\nu{}^{B}\mathcal{R}_{\rho\sigma}(S)\Big]$$

GR :	$\hat{P}_{\hat{A}}$	$\Leftrightarrow$	$\hat{M}_{\hat{A}\hat{B}}$
ESB gravity :	$H_A\,,\,P_{A'}\,,\,Z_A$	$\Leftrightarrow$	$M\,,\,G_{AA'}\,,\,J\,,\,S$

Gaugingt $\Leftrightarrow$ Action

Outline

Nonrelativitic String Theory

- Nonrelativistic String Theory can be defined independent of any limit of relativistic string theory

- String NC Geometry is to NR string theory what Riemannian geometry is to relativistic string theory

- NR target space actions exist!

Open Issues

- Does β-function calculation leads to consistent backgrounds?
- Nonrelativistic holography?

  Gopakumar, Bagchi (2009)
- Double Field Theory?

  S. M. Ko, C. Melby-Thompson, R. Meyer and J.-H. Park (2005)
- NR superstrings

Take Home Message

Nonrelativistic String Theory can be studied!

Gomis, Ooguri (2001)