Towards physical Yukawa couplings in heterotic model building

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Computations of physical Yukawa coupling is one of the most pressing problems in string phenomenology

Notoriously difficult because quite advanced knowledge of the geometry of the internal manifold is required

Any 4-dimensional string theory model must address this problem

Recall the particle spectrum of MSSM:

LH quarks $(\mathbf{3},\mathbf{2})_1 \quad Q$, RH *u* quarks $(\bar{\mathbf{3}},\mathbf{1})_{-4} \quad u$, RH *d* quarks $(\bar{\mathbf{3}},\mathbf{1})_2 \quad d$

LH leptons $(\mathbf{1},\mathbf{2})_{-3} \quad L$, RH electron $(\mathbf{1},\mathbf{1})_6 \quad e$

Higgs particles $(\mathbf{1},\mathbf{2})_{-3} \quad H$, $(\mathbf{1},\mathbf{2})_3 \quad \bar{H}$

RH neutrinos $(\mathbf{1},\mathbf{1})_0 \quad N$

Yukawa couplings: couplings of quarks and leptons to Higgses

up Yukawa coupling: $Q\bar{H}u$

down Yukawa couplings: $QHd,\ LHe$

In general: couplings among matter fields

I will discuss some approximate methods to compute YC in heterotic string theory on a Calabi-Yau manifold

The computations come in two steps:

1. Holomorphic YC, that is coupling in the superpotential

$$W = \lambda_{IJK} C^I C^J C^K$$

λ_{IJK} holomorphically depends on the moduli fields (complex structure moduli, vector bundle moduli). In some models λ_{IJK} can be explicitly computed

P. Candelas'88, S. Blesneag, EIB, P. Candelas, A. Lukas'15, S. Blesneag, EIB, A. Lukas'16

2. λ_{IJK} is NOT a physical YC. We have to canonically normalize the kinetic energy of matter fields

$$K = G_{IJ} C^I \bar{C}^J$$

G_{IJ} depends non-holomorphically on the moduli fields

In $D = 10$: g_{MN}, B_{MN}, A_M, fermions

$g_{MN} \to (g_{\mu\nu}, g_{a\bar{b}})$, $A_M \to (A_\mu, A_a, A_{\bar{b}})$

Supersymmetry conditions:

$g_{a\bar{b}}$ is the Ricci flat metric on a Calabi-Yau manifold X

A_a has to satisfy Hermitian Yang-Mills equations

$$F_{ab} = 0, \quad F_{\bar{a}\bar{b}} = 0, \quad g^{a\bar{b}} F_{a\bar{b}} = 0$$

A_a is a connection on a holomorphic stable vector bundle V

with structure group $G \in E_8$

$g_{a\bar{b}} \to$ Kahler moduli, complex structure moduli

$A_a \to$ vector bundle moduli

All moduli appear as 4-dimensional massless scalar fields

Physics in 4 dimensions is encoded in the geometry of the Calabi-Yau manifold and the vector bundle

No explicit solutions for the Ricci flat metric or Hermitian Yang-Mills connection are known

Matter fields originate from the $E_8 \times E_8$ gauge field and gaugino. Let G be the structure group of the vector bundle and H the low-energy gauge group. Matter multiplets can be read off from the branching

$$\mathbf{248} \to [(\mathrm{Adj}_G, 1) \oplus (1, \mathrm{Adj}_H) \oplus \bigoplus (R_G, R_H)]_{G\times H}$$

Matter multiplets transform as R_H under H

Geometrically: they correspond to **harmonic** $(0,1)$-forms ν transforming as R_G under G

Harmonic relative to the Ricci flat metric $\Longrightarrow$ hard to describe

Consider three reps. (R^i_G, R^i_H) such that $R^1_G \otimes R^2_G \otimes R^3_G$ contains a singlet. The holomorphic YC for the corresponding matter fields:

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3$$

Introduce basis forms ν_{iI} and denote

$$\lambda_{IJK} = \lambda(\nu_{1I}, \nu_{2J}, \nu_{3K})$$

Denote C_i^I the corresponding matter fields transforming as R_H^i. The superpotential is

$$W = \lambda_{IJK} C_1^I C_2^J C_3^K$$

Example: $H = SU(5)$, $G = SU(5)$

$$\mathbf{248} \to (\mathbf{24}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{24}) \oplus (\mathbf{5}, \mathbf{10}) \oplus (\bar{\mathbf{5}}, \overline{\mathbf{10}}) \oplus (\mathbf{10}, \bar{\mathbf{5}}) \oplus (\overline{\mathbf{10}}, \mathbf{5})$$

Up YC: $\mathbf{5} \cdot \mathbf{10} \cdot \mathbf{10}$, down YC: $\bar{\mathbf{5}} \cdot \bar{\mathbf{5}} \cdot \mathbf{10}$

These give rise to the standard model YC once we break $SU(5) \to SU(3) \times SU(2) \times U(1)$

Holomorphic YC depends only on the cohomology classes of the form $\nu_{iI} \Longrightarrow$ we can use any representatives, not necessarily harmonic forms relative to the Ricci flat metric

To describe CY: complete intersections in projective spaces

$\mathcal{A} = \mathcal{P}^{n_1} \times \ldots \times \mathcal{P}^{n_k}, \quad \sum n_i = d$

CY: $p_1 = 0, \ldots, p_m = 0, \ \dim \mathrm{CY} = d - m$

To describe a vector bundle: $V = \oplus_{a=1}^{n} L_a, \quad c_1(V) = 0$

If $n = 5$: $G = S(U(1)^5), \ H = SU(5) \times S(U(1)^5)$

V can be viewed as a limiting case of some vector bundle $\mathcal{V}$ with structure group $SU(5)$ when all bundle moduli are set to zero $\Longrightarrow$ approximation of small bundle moduli

Holomorphic YC can be computed by lifting all forms ν_i to $\mathcal{A}$. If $m = 1$ the forms $\hat{\nu}$ on $\mathcal{A}$ satisfy

$$\nu = \hat{\nu}|_X \,, \quad \bar{\partial}\hat{\nu} = p\hat{\omega} \,, \quad \bar{\partial}\hat{\omega} = 0$$

We can solve these equations on projective spaces

We can use any representatives for closed forms

$$\lambda = -\frac{1}{2\pi i} \int_{\mathcal{A}} d^4 z \wedge [\hat{\omega}_1 \wedge \hat{\nu}_2 \wedge \hat{\nu}_3 - \hat{\nu}_1 \wedge \hat{\omega}_2 \wedge \hat{\nu}_3 + \hat{\nu}_1 \wedge \hat{\nu}_2 \wedge \hat{\omega}_3]$$

Now let us discuss kinetic terms for matter fields

More complicated because non-holomorphic

Harmonic forms depend on the Ricci flat metric and Hermitian Yang-Mills connection

From dimensional reduction we find

$$G_{IJ} = \frac{1}{2\mathcal{V}} \int_X \nu_I \wedge \bar{*}_V(\nu_J), \quad K = G_{IJ} C^I \bar{C}^J$$

$\mathcal{V}$ is the CY volume, $\bar{*}_V$ is the Hodge star combined with a complex conjugation and the action of the Hermitian bundle metric H on V.

Universal Kahler moduli dependence:

$$G_{IJ} = \frac{3it^i t^j}{2d_{ijk}t^i t^j t^k} \Lambda_{ij,IJ} \quad \Lambda_{ij,IJ} = \int_X J_i \wedge J_j \wedge \nu_I \wedge H\bar{\nu}_J$$

$d_{ijk} = \int_X J_i \wedge J_j \wedge J_k$

$\Lambda_{ij,IJ}$ depends on complex structure and bundle moduli but not on Kahler moduli

It is impossible to compute normalization G_{IJ} exactly

We will use an approximation based on localization

When the gauge fluxes are substantially large the normalization integral is localized near a point

We can use approximately flat metric and flat connection

Similar ideas were used in F-theory: A. Font, L. Ibanez'09 L. Aparicio, A. Font, L. Ibanez, F. Marchesano'11, E. Palti'12

How to relate local calculations to global data: moduli, families?

Let's compute the analogue of the normalization integral on $\mathcal{P}^1$ and study its localization properties

We will use harmonic forms relative to the Fubini-Study metric

Introduce homogeneous coordinates Z_0, Z_1 and a local coordinate $z = Z_1/Z_0$. The Kahler form is

$$J = \frac{i}{2\pi(1+|z|^2)^2} dz \wedge d\bar{z}, \quad \int_{\mathcal{P}^1} J = 1$$

Let L be a line bundle on $\mathcal{P}^1$ with first Chern class k:

$$H = (1+|z|^2)^{-k}, \quad A_z = -\frac{k\bar{z}}{1+|z|^2}, \quad F = \bar{\partial}\partial \log H = -2\pi i k J$$

Harmonic bundle valued $(0,1)$-forms satisfy

$$\bar{\partial}\alpha = 0, \quad \partial(H * \alpha) = 0$$

The analogue of the normalization integral:

$$\langle \alpha, \beta \rangle = \int_{\mathcal{P}^1} \alpha \wedge *(H\bar{\beta})$$

If $k \geq -1$ there are no $(0,1)$-forms solving these equations

If $k \leq -2$ there are $(-k-1)$ independent solutions

$$\alpha_q = (1+|z|^2)^k \bar{z}^q d\bar{z}, \quad q = 0, 1, \ldots, -k-2$$

$$\langle \alpha_q, \alpha_p \rangle = \frac{2\pi q!}{(-k-1)(-k-2)\ldots(-k-1-q)} \delta_{qp}$$

Near $|z| = 0$:

$$J \to \frac{i}{2\pi} dz \wedge d\bar{z}, \quad H \to e^{-k|z|^2}, \quad \alpha_q \to e^{k|z|^2} \bar{z}^q d\bar{z}$$

α_q exponentially decays away from $|z| = 0$ for large $|k|$

$$\langle \alpha_q, \alpha_p \rangle_{loc} = \frac{2\pi q!}{(-k-1)^{q+1}} \delta_{qp}$$

$$\frac{\langle \alpha_q, \alpha_p \rangle_{loc}}{\langle \alpha_q, \alpha_p \rangle} = 1 - \mathcal{O}\Big(\frac{q^2}{-k-1}\Big)$$

Families corresponding to small values of q are well localized even if $|k|$ is not large

What if $q \sim |k|$? Then we can do localization near $z = \infty$.

For intermediate values of q localization may not work unless $|k|$ is large

This idea can be generalized to a product of projective spaces

Let X be a CY manifold of dimension three

Our goal is to determine the normalization of harmonic forms assuming localization indeed occurs

Let $U \subset X$ with local coordinates z_a. The Kahler form of the Ricci-flat metric approximately is

$$\mathcal{J} = \frac{i}{2\pi} \sum_{a=1}^{3} \beta_a dz_a \wedge d\bar{z}_a$$

On U we can approximate

$$\mathcal{H} = e^{-\sum_{a=1}^{3} K_a |z_a|^2}, \quad \mathcal{F} = \sum_{a=1}^{3} K_a dz_a \wedge d\bar{z}_a$$

Hermitian Yang-Mills equation: $\mathcal{J} \wedge \mathcal{J} \wedge \mathcal{F} = 0 \Longrightarrow$

$$\beta_1\beta_2 K_3 + \beta_1\beta_3 K_2 + \beta_2\beta_3 K_1 = 0$$

All $\beta_a > 0 \Longrightarrow$ not all K_a can have the same sign

Harmonic $(0,1)$-forms:

$$\bar{\partial}\nu = 0, \quad \mathcal{J} \wedge \mathcal{J} \wedge \partial(\mathcal{H}\nu) = 0$$

Let's assume $K_1 < 0, K_2, K_3 > 0$. The local solutions are of the form

$$\nu = e^{K_1|z_1|^2} P(\bar{z}_1, z_2, z_3) d\bar{z}_1$$

We can pick a basis of monomials: $\nu_{\vec{q}} = e^{K_1|z_1|^2} \bar{z}_1^{q_1} z_2^{q_2} z_3^{q_3} d\bar{z}_1$

The forms are well localized if $|K_1| \geq q_i$

$$M_{\vec{q}\vec{p}} = \langle \nu_{\vec{q}}, \nu_{\vec{p}} \rangle = \int_U \nu_{\vec{q}} \wedge *(\mathcal{H}\bar{\nu}_{\vec{p}})$$

$$\approx \frac{i\beta_2\beta_3\delta_{\vec{q}\vec{p}}}{4\pi} \prod_{a=1}^{3} \int_{\mathbf{C}} dz_a \wedge d\bar{z}_a |z_a|^{2q_a} e^{-|K_a||z_a|^2}$$

$$= 2\pi(\beta_2\beta_3)\delta_{\vec{q}\vec{p}} \prod_{a=1}^{3} q_a! |K_a|^{-q_a-1}$$

The integrand is localized for large $|K_a|$

Let X be a CY manifold, J_i a basis of harmonic $(0,1)$-forms, $J = \sum_i t^i J_i$ the Kahler form, $i = 1, \ldots, h^{1,1}(X)$. All these forms are harmonic relative to the Ricci flat metric. The cohomology class $[J_i]$ does not change as we change t_j. This means that $\frac{\partial J_i}{\partial t^j}$ is exact but not necessarily zero.

On a small patch U, $J \to \mathcal{J} = \sum_i t^i \mathcal{J}_i$

$J \wedge J \wedge J$ and $J_i \wedge J \wedge J$ are both harmonic top forms $\Longrightarrow$ they are proportional

$$J_i \wedge J \wedge J = c_i(t) J \wedge J \wedge J, \quad c_i(t) = \frac{d_{ijk} t^j t^k}{d_{ijk} t^i t^j t^k}$$

The same relation holds locally. Taking the local limit we can find the relation between $c_i(t)$ and β_a. Take $h^{1,1} = 2$

$$\mathcal{J}_1 = \frac{i}{2\pi} \sum_a \lambda_a dz_a \wedge d\bar{z}_a, \quad \mathcal{J}_2 = \frac{i}{2\pi} \sum_a dz_a \wedge d\bar{z}_a$$

$\mathcal{J} = \frac{i}{2\pi} \sum_{a=1}^{3} (\lambda_a t_1 + t_2) dz_a \wedge d\bar{z}_a$

$$c_1(t) = \frac{\sum_a \lambda_a \prod_{b\neq a}(\lambda_b t_1 + t_2)}{3\prod_c(\lambda_c t_1 + t_2)}, \; c_2(t) = \frac{\sum_a \prod_{b\neq a}(\lambda_b t_1 + t_2)}{3\prod_c(\lambda_c t_1 + t_2)}$$

We find λ_a in terms of global data

λ_a's seem to be t-independent

Example: $\mathcal{A} = \mathcal{P}^1 \times \mathcal{P}^3$

X is a hypersurface of degree $(2,4)$, $h^{1,1} = 2$. We define the Kahler parameters t_i by restricting the Kahler form from the ambient space $\quad \hat{J}|_X = t_1 \hat{J}_1|_X + t_2 \hat{J}_2|_X$

Neither $\hat{J}_1|_X, \hat{J}_2|_X, \hat{J}|_X$ are harmonic relative to the Ricci flat metric. The true harmonic forms J, J_i lie in the same cohomology classes as $\hat{J}|_X, \hat{J}_i|_X$

$$c_1(t) = \frac{2}{6t_1 + t_2}, \; c_2(t) = \frac{4t_1 + t_2}{t_2(6t_1 + t_2)}, \; \lambda_1 = 6, \; \lambda_2 = \lambda_3 = 0$$

$$\mathcal{J} = \frac{i}{2\pi}\Big[t_1 dz_1 \wedge d\bar{z}_1 + t_2\Big(\frac{1}{6} dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2 + dz_3 \wedge d\bar{z}_3\Big)\Big]$$

Let us focus on a specific open set $U = \hat{U}|_X$, where $\hat{U} \subset \mathcal{A}$ containing $\hat{z}_\alpha = 0$. If it is sufficiently small the equation for X is $p = \sum_{\alpha=1}^{4} p_\alpha \hat{z}_\alpha + \mathcal{O}(\hat{z}^2) = 0$

Redefine coordinates (fixed complex structure): $\hat{z}_4 = a\hat{z}_1$

Require $\hat{J}_i|_U = \mathcal{J}_i \Longrightarrow a = 1/\sqrt{6}$

Let us consider a line bundle L with first Chern class $c_1(L) = k_1\hat{J}_1 + k_2\hat{J}_2, \quad k_1 \leq -2, k_2 > 0$:

$$\mathcal{H} = e^{-(k_1+k_2/6)|z_1|^2 - k_2|z_2|^2 - k_3|z_3|^2}, \quad \mathcal{F} = -2\pi i(k_1\mathcal{J}_1 + k_2\mathcal{J}_2)$$

We can relate harmonic forms on $\mathcal{A}$ to local solutions

The normalization integral:

$$G_{IJ} = \frac{N_{IJ}}{6t_1 + t_2}, \quad N_{0,0} = 3\pi\frac{|k_1 + k_2/6|}{k_2^2}$$

The Kahler moduli dependence is fixed by supersymmetry

It does not depend on complex structure because of our approximation.

Conclusion

- Physical YC: holomorphic YC, normalization
- Holomorphic YC can be computed in a large class of models
- Normalization can be computed using localization if the gauge fluxes are large
- It is possible to relate local and global quantities
- Large fluxes tend to imply many generations $\Longrightarrow$ tension between localization and phenomenology