COSMOLOGICAL CONSTANT AS A CONSERVED CHARGE

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Black holes in asymptotically (A)dS space in many respects behave as thermodynamic systems under pressure P and having a volume V (D. Kastor et. al (2009), B. Dolan (2011),...)

- Black hole chemistry (D. Kubiznak et. al (2016))

Generalized first law of black hole thermodynamics

$$T\delta S = \delta M - \Omega\delta J - \Phi dQ - V\delta P$$

where T is Hawking temperature, S - entropy, M - mass, J and Φ are chemical potentials, Q - electric charge, V - thermodynamic volume and P - pressure. Pressure is defined by

$$P = \frac{\Lambda}{8\pi G}$$

Motivation

Examples: thermodynamical volume of

- nonrotating black hole $V = \frac{4}{3}\pi r_H^2$
- Kerr-AdS black hole $V = \frac{4\pi\left(r_H^2 + r_H a^2 + \frac{GMa^2}{1+\Lambda a^2/3}\right)}{3(1+\Lambda a^2/3)}$

What could be a rigorous definition for the V? (some earlier works: M. Cvetic et. al (2011), S. Hyun et. al (2017))

Cosmological constant as a parameter in Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

Varying such an external parameter in the first law is not entirely legal. Is it?

Cosmological constant from a top–form gauge field

Cosmological constant as on-shell value of a top-form (A. Aurilia et. al (1980), S. Hawking (1984)):

$$S = \int d^D x \, (\mathcal{L}_g + \mathcal{L}_M) + \frac{1}{16\pi G} \int *F F,$$

where F is a top form

$$F = \frac{1}{D!} F_{\mu_1 \dots \mu_D} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_D}, \qquad F = dA.$$

Equations of motion

$$d * F = 0 \Longrightarrow \nabla_\mu F^{\mu \dots \mu_D} = 0$$

imply that

$$F_{\mu_1 \dots \mu_D} = c \sqrt{-g} \epsilon_{\mu_1 \dots \mu_D}$$

where c is an integration constant. Substituting it into the action, one finds

$$S = \int d^D x \, (\mathcal{L}_g + \mathcal{L}_M) + \frac{1}{16\pi G} \int d^D x \sqrt{-g} \Lambda,$$

where $\Lambda = c^2$.

Cosmological constant as a conserved charge

Gauge symmetry

$$F = dA, \qquad A \to A + d\lambda, \qquad \lambda = \frac{1}{(D-2)!}\lambda_{\mu_1 \dots \mu_{D-2}} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_{D-2}}$$

Considering the set of closed gauge transformation generators $d\lambda = 0$, in which λ is normalized by the non-zero constant $\oint_S \lambda$. Corresponding Noether charge is

$$C = \frac{1}{4\pi G}\oint_S (d^{D-2}x)_{\mu\nu} Q^{\mu\nu}, \qquad Q^{\mu\nu} = \frac{1}{(D-2)!} F^{\mu\nu\rho_3 \dots \rho_D} \lambda_{\rho_3 \dots \rho_D}$$

Taking onshell value of F, one finds

$$C = \frac{c}{4\pi G}, \qquad or \qquad \Lambda = (4\pi G)^2 C^2$$

Generalized first law

Generalized first law via covariant phase space formalism (J. Lee, R. Wald (1992); R. Wald, A. Zoupas (2000); G. Barnich, F. Brandt (2002)):
Denote generators of diffeomorphism and gauge transformations by $\eta = \{\xi^\mu, \lambda\}$, where ξ^μ corresponds to diffeomorphism, while λ is generator of the gauge transformations of A. Dynamical fields transform as

$$\delta_\eta g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}, \qquad \delta_\eta A = \mathcal{L}_\xi A + d\lambda$$

where $\mathcal{L}_\xi$ is Lie derivative. In the Lee-Wald formulation, a charge variation can be attributed to an arbitrary η

$$\delta Q_\eta = \oint_S \boldsymbol{k}_\eta, \qquad \boldsymbol{k}_\eta = \frac{\sqrt{-g}\left(k_\epsilon^{E\mu\nu} + k_\epsilon^{F\mu\nu}\right)}{2!(D-2)!}\epsilon_{\mu\nu\mu_1\ldots\mu_{D-2}} dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_{D-2}},$$

where

$$k_\eta^{E\mu\nu} = \frac{1}{8\pi G}\left[\xi^\nu\nabla^\mu h^\alpha_{\ \alpha} - \xi^\nu\nabla_\rho h^{\mu\rho} + \xi_\rho\nabla^\nu h^{\mu\rho} + \frac{1}{2}h^\alpha_{\ \alpha}\nabla^\nu\xi^\mu - h^{\rho\nu}\nabla_\rho\xi^\mu\right]$$

$$k_\eta^{F\mu\nu} = \frac{1}{4\pi G(D-2)!}\Big[\Big(\frac{-h^\alpha_{\ \alpha}}{2}F^{\mu\nu\rho_3\ldots\rho_D} + 2h^{\mu\beta}F_\beta^{\ \nu\rho_3\ldots\rho_D} - \delta F^{\mu\nu\rho_3\ldots\rho_D}\Big)(\xi^\sigma A_{\sigma\rho_3\ldots\rho_D}$$

$$+\lambda_{\rho_3\ldots\rho_D}) - F^{\mu\nu\rho_3\ldots\rho_D}\xi^\sigma\delta A_{\sigma\rho_3\ldots\rho_D} + \lambda_{\beta\rho_4\ldots\rho_D}) + \frac{2}{D-1}F^{\mu\rho_2\ldots\rho_D}\xi^\nu\delta A_{\rho_2\ldots\rho_D}\Big]$$

For our purposes η should generate exact symmetry.

Generalized first law

Define conjugate chemical potential

$$\Theta_H \equiv \oint_H \zeta_H \cdot A$$

where H is for horizon and ζ_h is a horizon generating Killing vector. Note the analogy with electric chemical potential $\Phi_H = (\zeta_H \cdot A_M)|_H$, where A_M is Maxwell field. Next, define a generator

$$\eta = \frac{2\pi}{\kappa_H}\{\zeta_H^\mu, -\Phi_H, -\Theta_H \lambda\}$$

Now we can derive the generalized first law of black hole thermodynamics

$$T_H \delta S = \delta M - \Omega_H^i \delta J_i - \Phi_H \delta Q - \Theta_H \delta C$$

Relation of the chemical potential Θ_H to thermodynamic volume

$$\Theta_H \delta C = V \delta P \Rightarrow V = -\frac{\Theta_H}{\sqrt{|\Lambda|}}$$

Example: Kerr black hole

Kerr-Newman-(A)dS solution describes rotating black hole in asymptotically (A)dS space. In addition to the metric and Maxwell field define

$$A = -\frac{\sqrt{|\Lambda|}\sin\theta(r^3 + 3ra^2\cos^2\theta + \frac{Gma^2}{1+\Lambda a^2/3})}{3(1+\Lambda a^2/3)} dt \wedge d\theta \wedge d\phi$$

Corresponding chemical potential is

$$\Theta_H = \frac{\sqrt{\Lambda}4\pi(r_H^3 + r_H a^2 + \frac{Gma^2}{\Xi})}{3\Xi}.$$

Together with the other chemical potentials

$$\kappa_H = \frac{r_H(1 - \frac{\Lambda a^2}{3} - \Lambda r_H^2 - \frac{a^2}{r_H^2})}{2(r_H^2 + a^2)}, \qquad \Omega_H = \frac{a(1 - \frac{\Lambda r_H^2}{3})}{r_H^2 + a^2}$$

and conserved charges

$$M = \frac{m}{\Xi^2}, \quad J = \frac{ma}{\Xi^2}, \quad S = \frac{\pi(r_H^2 + a^2)}{G\Xi}, \quad C = \frac{\pm\sqrt{|\Lambda|}}{4\pi G}$$

they satisfy first law of thermodynamics

$$T_H \delta S = \delta M - \Omega_H \delta J + \Theta_H \delta C$$