On locality in HS theory

S. Didenko
(based on arXiv:1807.00001 with O.A Gelfond, A.V. Korybut and M. A. Vasiliev)

July 31, 2018 Tomsk

Plan

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)
- Vasiliev theory is dual to vectorial CFT models

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)
- Vasiliev theory is dual to vectorial CFT models

$$S_{HS}[\phi] \sim \langle J \dots J \rangle_{CFT}$$

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)
- Vasiliev theory is dual to vectorial CFT models

$$S_{HS}[\phi] \sim \langle J \dots J \rangle_{CFT}$$

- Giombi and Yin tests from equations of motion: substantial piece of evidence that many of $3pt$ functions match.

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)
- Vasiliev theory is dual to vectorial CFT models

$$S_{HS}[\phi] \sim \langle J \dots J \rangle_{CFT}$$

- Giombi and Yin tests from equations of motion: substantial piece of evidence that many of $3pt$ functions match.
- Generic structure of $3pt$-correlators (Maldacena, Zhiboedov)

$$\langle JJJ \rangle = \cos^2\phi \langle JJJ \rangle_b + \sin^2\phi \langle JJJ \rangle_f + \frac{1}{2}\sin(2\phi)\langle JJJ \rangle_o$$

Locality

- GY: Interaction vertices are sometimes so nonlocal that produce infinities in correlation functions.

Locality

- GY: Interaction vertices are sometimes so nonlocal that produce infinities in correlation functions. Some correlators can be reached via analytic continuation, but some happened to be inconsistent with crossing.

Locality

- GY: Interaction vertices are sometimes so nonlocal that produce infinities in correlation functions. Some correlators can be reached via analytic continuation, but some happened to be inconsistent with crossing. Free boson and free fermion were captured.

Locality

- GY: Interaction vertices are sometimes so nonlocal that produce infinities in correlation functions. Some correlators can be reached via analytic continuation, but some happened to be inconsistent with crossing. Free boson and free fermion were captured. Parity odd correlators $\langle J_0 J_{s_1} J_{s_2} \rangle$ were accessible only in one channel.

Locality

- GY: Interaction vertices are sometimes so nonlocal that produce infinities in correlation functions. Some correlators can be reached via analytic continuation, but some happened to be inconsistent with crossing. Free boson and free fermion were captured. Parity odd correlators $\langle J_0 J_{s_1} J_{s_2} \rangle$ were accessible only in one channel.
- Locality

$$S = \int \phi \Box \phi + D..D\phi D..D\phi D..D\phi + D..D\phi D..D\phi D..D\phi D..D\phi$$

Locality

- GY: Interaction vertices are sometimes so nonlocal that produce infinities in correlation functions. Some correlators can be reached via analytic continuation, but some happened to be inconsistent with crossing. Free boson and free fermion were captured. Parity odd correlators $\langle J_0 J_{s_1} J_{s_2} \rangle$ were accessible only in one channel.
- Locality

$$S = \int \phi\Box\phi + D..D\phi D..D\phi D..D\phi + D..D\phi D..D\phi D..D\phi D..D\phi$$

At cubic order the interaction is known to have local form.

(Non)locality

- Vasiliev's approach to HS dynamics

(Non)locality

- Vasiliev's approach to HS dynamics

$$dω(Y|x) = -ω * ω + \Upsilon(ω, ω, C \dots C)\,,$$
$$dC(Y|x) = -[ω, C]_* + \Upsilon(ω, C \dots C)\,.$$

(Non)locality

- Vasiliev's approach to HS dynamics

$$
\begin{aligned}
d\omega(Y|x) &= -\omega * \omega + \Upsilon(\omega, \omega, C \dots C)\,,\\
dC(Y|x) &= -[\omega, C]_* + \Upsilon(\omega, C \dots C)\,.
\end{aligned}
$$

$\omega(Y|x)$ - encodes HS potentials (polynomial in Y for a given spin)

(Non)locality

- Vasiliev's approach to HS dynamics

$$d\omega(Y|x) = -\omega * \omega + \Upsilon(\omega, \omega, C \dots C)\,,$$
$$dC(Y|x) = -[\omega, C]_* + \Upsilon(\omega, C \dots C)\,.$$

$\omega(Y|x)$ - encodes HS potentials (polynomial in Y for a given spin)
$C(Y|x)$ - encodes HS on-shell derivatives (unbounded in Y for a given spin)

(Non)locality

- Vasiliev's approach to HS dynamics

$$d\omega(Y|x) = -\omega * \omega + \Upsilon(\omega, \omega, C \dots C)\,,$$
$$dC(Y|x) = -[\omega, C]_* + \Upsilon(\omega, C \dots C)\,.$$

$\omega(Y|x)$ - encodes HS potentials (polynomial in Y for a given spin)
$C(Y|x)$ - encodes HS on-shell derivatives (unbounded in Y for a given spin)

- Origin of potential non-localities is the star-product

(Non)locality

- Vasiliev's approach to HS dynamics

$$dω(Y|x) = -ω * ω + Υ(ω, ω, C \dots C)\,,$$
$$dC(Y|x) = -[ω, C]_* + Υ(ω, C \dots C)\,.$$

$ω(Y|x)$ - encodes HS potentials (polynomial in Y for a given spin)
$C(Y|x)$ - encodes HS on-shell derivatives (unbounded in Y for a given spin)

- Origin of potential non-localities is the star-product

$$f(y) * g(y) = f(y) e^{i\epsilon^{\alpha\beta}\overleftarrow{\partial_\alpha}\overrightarrow{\partial_\beta}} g(y)\,,$$

(Non)locality

- Vasiliev's approach to HS dynamics

$$d\omega(Y|x) = -\omega * \omega + \Upsilon(\omega, \omega, C \dots C)\,,$$
$$dC(Y|x) = -[\omega, C]_* + \Upsilon(\omega, C \dots C)\,.$$

$\omega(Y|x)$ - encodes HS potentials (polynomial in Y for a given spin)
$C(Y|x)$ - encodes HS on-shell derivatives (unbounded in Y for a given spin)

- Origin of potential non-localities is the star-product

$$f(y) * g(y) = f(y) e^{i\epsilon^{\alpha\beta}\overleftarrow{\partial_\alpha}\overrightarrow{\partial_\beta}} g(y)\,,$$

- Structure of perturbation theory

$$d_Z \mathrm{Field}(Z, Y) = I(Z, Y)$$

(Non)locality

- Vasiliev's approach to HS dynamics

$$d\omega(Y|x) = -\omega * \omega + \Upsilon(\omega, \omega, C \dots C)\,,$$
$$dC(Y|x) = -[\omega, C]_* + \Upsilon(\omega, C \dots C)\,.$$

$\omega(Y|x)$ - encodes HS potentials (polynomial in Y for a given spin)
$C(Y|x)$ - encodes HS on-shell derivatives (unbounded in Y for a given spin)

- Origin of potential non-localities is the star-product

$$f(y) * g(y) = f(y) e^{i\epsilon^{\alpha\beta}\overleftarrow{\partial_\alpha}\overrightarrow{\partial_\beta}} g(y)\,,$$

- Structure of perturbation theory

$$d_Z \text{Field}(Z, Y) = I(Z, Y)\,, \quad \text{Field}(Z, Y) = d^{-1} I(Z, Y) + \text{H}(Y)$$

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices.

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices. Breakdown of predictivity of Vasiliev theory – *any* coefficients result from nonlocal field redefinitions.

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices. Breakdown of predictivity of Vasiliev theory – *any* coefficients result from nonlocal field redefinitions.Wrong phase dependence (inconsistent with Maldacena-Zhiboedov)

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices. Breakdown of predictivity of Vasiliev theory – *any* coefficients result from nonlocal field redefinitions.Wrong phase dependence (inconsistent with Maldacena-Zhiboedov)
- Vasiliev+Gelfond: At qubic order there is a solution that leads to local vertices.

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices. Breakdown of predictivity of Vasiliev theory – *any* coefficients result from nonlocal field redefinitions.Wrong phase dependence (inconsistent with Maldacena-Zhiboedov)
- Vasiliev+Gelfond: At qubic order there is a solution that leads to local vertices.The dependence on phase is correct.

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices. Breakdown of predictivity of Vasiliev theory – *any* coefficients result from nonlocal field redefinitions.Wrong phase dependence (inconsistent with Maldacena-Zhiboedov)
- Vasiliev+Gelfond: At qubic order there is a solution that leads to local vertices.The dependence on phase is correct.
- S.D.+Vasiliev Coupling constants are in perfect agreement with the AdS/CFT expectations.

(Non)locality

- Non-localities in ω-sector (N. Boulanger, P. Kessel, E.D. Skvortsov, M. Taronna)– Infinite coefficients in front of cubic vertices. Breakdown of predictivity of Vasiliev theory – *any* coefficients result from nonlocal field redefinitions.Wrong phase dependence (inconsistent with Maldacena-Zhiboedov)
- Vasiliev+Gelfond: At qubic order there is a solution that leads to local vertices.The dependence on phase is correct.
- S.D.+Vasiliev Coupling constants are in perfect agreement with the AdS/CFT expectations.
- N.G. Misuna Coupling coefficients are in agreement with those of holographically reconstructed (S. Sleight, M. Taronna)

Goals and summary

GOALS:

- Develop perturbation theory for HS equations that is consistent with locality.

Goals and summary

GOALS:

- Develop perturbation theory for HS equations that is consistent with locality.
- Gelfond+Vasiliev: Shifted homotopies and locality theorem.

Goals and summary

GOALS:

- Develop perturbation theory for HS equations that is consistent with locality.
- Gelfond+Vasiliev: Shifted homotopies and locality theorem.
- Calculate explicitly some lower order interaction vertices.

Goals and summary

RESULTS:

- Remarkable formulas relating star-product and shifted homotopies found.

Goals and summary

RESULTS:

- Remarkable formulas relating star-product and shifted homotopies found.
- Vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, C, C)$ are found to have (ultra)local form for homotopies compatible with locality theorem.

Goals and summary

RESULTS:

- Remarkable formulas relating star-product and shifted homotopies found.
- Vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, C, C)$ are found to have (ultra)local form for homotopies compatible with locality theorem.
- A one parameter family of such homotopies was observed to lead identically equivalent vertices.

Vasiliev equations

Vasiliev equations in $d = 4$

$$
\begin{aligned}
&\mathrm{d}_x W + W * W = 0\,,\\
&\mathrm{d}_x S + [W, S]_* = 0\,,\\
&\mathrm{d}_x B + [W, B]_* = 0\,,\\
&S * S = -i\theta_\alpha \theta^\alpha + i\eta B * \gamma + c.c.\,,\\
&[S, B]_* = 0\,.
\end{aligned}
$$

$$\gamma = e^{iz_\alpha y^\alpha}\theta^\beta \theta_\beta$$

Vasiliev equations

Vasiliev equations in $d = 4$

$$
\begin{aligned}
&\mathrm{d}_x W + W * W = 0\,,\\
&\mathrm{d}_x S + [W, S]_* = 0\,,\\
&\mathrm{d}_x B + [W, B]_* = 0\,,\\
&S * S = -i\theta_\alpha\theta^\alpha + i\eta B * \gamma + c.c.\,,\\
&[S, B]_* = 0\,.
\end{aligned}
$$

$$
\gamma = e^{iz_\alpha y^\alpha}\theta^\beta\theta_\beta
$$

$$
W(Z, Y|x) = \omega(Y|x) + \dots\,, \quad B(Z, Y|x) = C(Y|x) + \dots
$$

Perturbation theory

Vacuum

$$S^0 = z_\alpha \theta^\alpha\,, \quad B^0 = 0\,, \quad W^0 = \omega(Y|x)$$

$$\mathrm{d}_x \omega + \omega * \omega = 0\,.$$

Perturbation theory

Vacuum

$$S^0 = z_\alpha \theta^\alpha \,, \quad B^0 = 0 \,, \quad W^0 = \omega(Y|x)$$

$$\mathrm{d}_x \omega + \omega * \omega = 0 \,.$$

Higher orders

$$[S^0, f] \sim d_z f$$

Perturbation theory

Vacuum

$$S^0 = z_\alpha \theta^\alpha \,, \quad B^0 = 0 \,, \quad W^0 = \omega(Y|x)$$

$$\mathrm{d}_x \omega + \omega * \omega = 0 \,.$$

Higher orders

$$[S^0, f] \sim d_z f$$

Common equation to solve:

$$d_z f(z, y; \theta) = J(z, y; \theta) \,, \quad f = \Delta_0 J + d_z \epsilon + H(y) \,,$$

Perturbation theory

Vacuum

$$S^0 = z_\alpha \theta^\alpha\,, \quad B^0 = 0\,, \quad W^0 = \omega(Y|x)$$

$$\mathrm{d}_x \omega + \omega * \omega = 0\,.$$

Higher orders

$$[S^0, f] \sim d_z f$$

Common equation to solve:

$$d_z f(z, y; \theta) = J(z, y; \theta)\,, \quad f = \Delta_0 J + d_z \epsilon + H(y)\,,$$

Conventional homotopy:

$$\Delta_0 J = z^\alpha \frac{\partial}{\partial \theta^\alpha} \int_0^1 \frac{dt}{t} J(tz, y; t\theta)$$

Shifted homotopy

Alternative way of writing solution

Instead of applying homotopy to z, one can apply it to $z - q$, where q is an arbitrary z-independent parameter.

Shifted homotopy

Alternative way of writing solution

Instead of applying homotopy to z, one can apply it to $z - q$, where q is an arbitrary z-independent parameter.

$$f = \Delta_q J = (z^\alpha - q^\alpha)\frac{\partial}{\partial\theta^\alpha}\int_0^1 \frac{dt}{t} J(tz + (1-t)q, y; t\theta)$$

Shifted homotopy

Alternative way of writing solution

Instead of applying homotopy to z, one can apply it to $z - q$, where q is an arbitrary z-independent parameter.

$$f = \Delta_q J = (z^\alpha - q^\alpha)\frac{\partial}{\partial\theta^\alpha}\int_0^1 \frac{dt}{t} J(tz + (1-t)q, y; t\theta)$$

q may depend on y_α or ∂_α in a Lorentz covariant fashion.

Shifted homotopy

Alternative way of writing solution

Instead of applying homotopy to z, one can apply it to $z-q$, where q is an arbitrary z-independent parameter.

$$f=\Delta_q J=(z^\alpha-q^\alpha)\frac{\partial}{\partial\theta^\alpha}\int_0^1\frac{dt}{t}J(tz+(1-t)q,y;t\theta)$$

q may depend on y_α or ∂_α in a Lorentz covariant fashion.

$$q_\alpha=ay_\alpha+v_1\partial_{1\alpha}+\dots v_n\partial_{n\alpha}$$

Homotopy properties

- Resolution of identity

$$\{d_z, \Delta_q\} = 1 - h_q\,, \qquad h_q f(z, y; \theta) = f(q, y; 0)$$

Homotopy properties

- Resolution of identity

$$\{d_z, \Delta_q\} = 1 - h_q\,, \qquad h_q f(z, y; \theta) = f(q, y; 0)$$

- anti-commutativity

$$\Delta_p \Delta_q = -\Delta_q \Delta_p\,, \qquad h_p \Delta_q = -h_q \Delta_p$$

Homotopy properties

- Resolution of identity

$$\{d_z, \Delta_q\} = 1 - h_q\,, \qquad h_q f(z, y; \theta) = f(q, y; 0)$$

- anti-commutativity

$$\Delta_p \Delta_q = -\Delta_q \Delta_p\,, \qquad h_p \Delta_q = -h_q \Delta_p$$

$$f_{\alpha\beta}(z, y; \theta) := \frac{\partial^2}{\partial\theta^\alpha \partial\theta^\beta} f(z, y; \theta)\,.$$

Homotopy properties

- Resolution of identity

$$\{d_z, \Delta_q\} = 1 - h_q\,, \qquad h_q f(z, y; \theta) = f(q, y; 0)$$

- anti-commutativity

$$\Delta_p \Delta_q = -\Delta_q \Delta_p\,, \qquad h_p \Delta_q = -h_q \Delta_p$$

$$f_{\alpha\beta}(z, y; \theta) := \frac{\partial^2}{\partial\theta^\alpha \partial\theta^\beta} f(z, y; \theta)\,.$$

$$\Delta_p \Delta_q f(z, y; \theta) = \int_{[0,1]^3} \mathrm{d}^3\tau\, \delta(1 - \tau_1 - \tau_2 - \tau_3)(z^\beta - p^\beta)(z^\alpha - q^\alpha) f_{\alpha\beta}(\tau_3 z + \tau_1 p + \tau_2 q, y; \tau_3\theta)$$

Homotopy properties

- degeneracy relation

$$h_{(\mu+1)p-\mu q}\Delta_p\Delta_q = 0\,, \qquad \forall\mu\in\mathbb{C}$$

Homotopy properties

- degeneracy relation

$$h_{(\mu+1)p-\mu q}\Delta_p\Delta_q = 0\,, \qquad \forall\mu\in\mathbb{C}$$

- γ-property

$$(\Delta_c\Delta_b - \Delta_c\Delta_a + \Delta_b\Delta_a)\gamma = h_c\Delta_b\Delta_a\gamma$$

Homotopy properties

- degeneracy relation

$$h_{(\mu+1)p-\mu q}\Delta_p\Delta_q = 0\,, \qquad \forall \mu \in \mathbb{C}$$

- γ-property

$$(\Delta_c\Delta_b - \Delta_c\Delta_a + \Delta_b\Delta_a)\gamma = h_c\Delta_b\Delta_a\gamma$$

- triangle identity

$$h_d\Delta_c\Delta_b\gamma + h_d\Delta_b\Delta_a\gamma = h_c\Delta_b\Delta_a\gamma + h_d\Delta_c\Delta_a\gamma$$

Homotopy properties

- degeneracy relation

$$h_{(\mu+1)p-\mu q}\Delta_p\Delta_q = 0\,, \qquad \forall\mu\in\mathbb{C}$$

- γ-property

$$(\Delta_c\Delta_b - \Delta_c\Delta_a + \Delta_b\Delta_a)\gamma = h_c\Delta_b\Delta_a\gamma$$

- triangle identity

$$h_d\Delta_c\Delta_b\gamma + h_d\Delta_b\Delta_a\gamma = h_c\Delta_b\Delta_a\gamma + h_d\Delta_c\Delta_a\gamma$$

- homogeneity property

$$h_{a+\alpha y}\Delta_{b+\alpha y}\Delta_{c+\alpha y}\gamma = h_a\Delta_b\Delta_c\gamma\,, \qquad \forall\alpha\in\mathbb{C}$$

star-product and homotopies

$$\Delta_{q+\alpha y}(f(y)*\phi(z,y;\theta)) = f(y)*\Delta_{q+(1-\alpha)p+\alpha y}\phi(z,y;\theta)\,,$$

star-product and homotopies

$$\Delta_{q+\alpha y}(f(y) * \phi(z, y; \theta)) = f(y) * \Delta_{q+(1-\alpha)p+\alpha y}\phi(z, y; \theta)\,,$$

where

$$p_\alpha f(y) \equiv f(y) p_\alpha := i\frac{\partial}{\partial y^\alpha} f(y)$$

star-product and homotopies

$$\Delta_{q+\alpha y}(f(y)*\phi(z,y;\theta)) = f(y)*\Delta_{q+(1-\alpha)p+\alpha y}\phi(z,y;\theta)\,,$$

where

$$p_\alpha f(y) \equiv f(y)p_\alpha := i\frac{\partial}{\partial y^\alpha}f(y)$$

$$\mathrm{d}_z g = f(y)*\phi(z,y;\theta) \quad \Rightarrow \quad g=\Delta_a(f*\phi) \quad \text{or} \quad g = f*\Delta_b\phi$$

star-product and homotopies

$$\Delta_{q+\alpha y}(f(y)*\phi(z,y;\theta)) = f(y)*\Delta_{q+(1-\alpha)p+\alpha y}\phi(z,y;\theta)\,,$$

where

$$p_\alpha f(y) \equiv f(y)p_\alpha := i\frac{\partial}{\partial y^\alpha}f(y)$$

$$\mathrm{d}_z g = f(y)*\phi(z,y;\theta) \quad \Rightarrow \quad g=\Delta_a(f*\phi) \quad \text{or} \quad g=f*\Delta_b\phi$$

Analogously,

$$\Delta_{q+\alpha y}(\phi(z,y;\theta)*f(y)) = \Delta_{q+(1+\alpha)p+\alpha y}\phi(z,y;\theta)*f(y)$$

star-product and homotopies

$$\Delta_{q+\alpha y}(f(y)*\phi(z,y;\theta)) = f(y)*\Delta_{q+(1-\alpha)p+\alpha y}\phi(z,y;\theta)\,,$$

where

$$p_\alpha f(y) \equiv f(y)p_\alpha := i\frac{\partial}{\partial y^\alpha}f(y)$$

$$\mathrm{d}_z g = f(y)*\phi(z,y;\theta) \quad \Rightarrow \quad g=\Delta_a(f*\phi) \quad \text{or} \quad g = f*\Delta_b\phi$$

Analogously,

$$\Delta_{q+\alpha y}(\phi(z,y;\theta)*f(y)) = \Delta_{q+(1+\alpha)p+\alpha y}\phi(z,y;\theta)*f(y)$$

$$\Delta_q\gamma * f(y) = f(y)*\Delta_{q+2p}\gamma$$

Locality theorem

- Structure theorem (O.A. Gelfond). There is the *even* one-form sector of HS equations (W and S) that is respected by Δ_{s_e} – homotopies and the *odd* zero-form one (B) respected by Δ_{s_o}

Locality theorem

- Structure theorem (O.A. Gelfond). There is the *even* one-form sector of HS equations (W and S) that is respected by Δ_{s_e} – homotopies and the *odd* zero-form one (B) respected by Δ_{s_o}

$$s_e = w_1 p_1 + \dots w_n p_n + \mu y \,, \quad \sum_{i=1}^{n} (-)^i w_i = -\mu$$

Locality theorem

- Structure theorem (O.A. Gelfond). There is the *even* one-form sector of HS equations (W and S) that is respected by Δ_{s_e} – homotopies and the *odd* zero-form one (B) respected by Δ_{s_o}

$$s_e = w_1 p_1 + \dots w_n p_n + \mu y\,, \quad \sum_{i=1}^{n} (-)^i w_i = -\mu$$

$$s_o = v_1 p_1 + \dots v_n p_n + \mu y\,, \quad \sum_{i=1}^{n} (-)^i v_i = 1$$

Locality theorem

- Structure theorem (O.A. Gelfond). There is the *even* one-form sector of HS equations (W and S) that is respected by Δ_{s_e} – homotopies and the *odd* zero-form one (B) respected by Δ_{s_o}

$$s_e = w_1 p_1 + \dots w_n p_n + \mu y\,, \quad \sum_{i=1}^{n} (-)^i w_i = -\mu$$

$$s_o = v_1 p_1 + \dots v_n p_n + \mu y\,, \quad \sum_{i=1}^{n} (-)^i v_i = 1$$

- Pfaffian locality theorem (Gelfond+Vasiliev). The degree of non-locality of zero-form HS vertices gets reduced on the homotopies prescribed by the structure theorem.

Locality theorem

- Structure theorem (O.A. Gelfond). There is the *even* one-form sector of HS equations (W and S) that is respected by Δ_{s_e} – homotopies and the *odd* zero-form one (B) respected by Δ_{s_o}

$$s_e = w_1 p_1 + \dots w_n p_n + \mu y\,, \quad \sum_{i=1}^{n} (-)^i w_i = -\mu$$

$$s_o = v_1 p_1 + \dots v_n p_n + \mu y\,, \quad \sum_{i=1}^{n} (-)^i v_i = 1$$

- Pfaffian locality theorem (Gelfond+Vasiliev). The degree of non-locality of zero-form HS vertices gets reduced on the homotopies prescribed by the structure theorem.

$$\det P_{ij} = 0\,, \quad \exp P_{ij}\partial^i_\alpha \partial^{j\alpha} \quad \Rightarrow \quad \text{measure of non-locality}$$

$\Upsilon(\omega, \omega, C)$ – vertex

$$d_z S_1 = -\frac{\eta}{2} C * \gamma \quad \Rightarrow \quad S_1 = -\frac{\eta}{2}\Delta_0(C * \gamma) = -\frac{\eta}{2} C * \Delta_p \gamma\,,$$

$\Upsilon(\omega,\omega,C)$ – vertex

$$d_z S_1 = -\frac{\eta}{2} C * \gamma \quad \Rightarrow \quad S_1 = -\frac{\eta}{2}\Delta_0 (C * \gamma) = -\frac{\eta}{2} C * \Delta_p \gamma\,,$$

$$W_1 = \frac{1}{2i}\Delta_0 (\mathrm{d}_x S_1 + \omega * S_1 + S_1 * \omega)\,,$$

$\Upsilon(\omega,\omega,C)$ – vertex

$$d_z S_1 = -\frac{\eta}{2} C * \gamma \quad \Rightarrow \quad S_1 = -\frac{\eta}{2}\Delta_0(C*\gamma) = -\frac{\eta}{2} C * \Delta_p \gamma\,,$$

$$W_1 = \frac{1}{2i}\Delta_0(\mathrm{d}_x S_1 + \omega * S_1 + S_1 * \omega)\,,$$

$$\mathrm{d}_x C + [\omega, C]_* = 0\,,$$

$\Upsilon(\omega, \omega, C)$ – vertex

$$d_z S_1 = -\frac{\eta}{2} C * \gamma \quad \Rightarrow \quad S_1 = -\frac{\eta}{2}\Delta_0(C * \gamma) = -\frac{\eta}{2} C * \Delta_p \gamma\,,$$

$$W_1 = \frac{1}{2i}\Delta_0(\mathrm{d}_x S_1 + \omega * S_1 + S_1 * \omega)\,,$$

$$\mathrm{d}_x C + [\omega, C]_* = 0\,,$$

$$W_1 = -\frac{\eta}{4i}\left(C * \omega * \Delta_{p+t}\Delta_{p+2t}\gamma - \omega * C * \Delta_{p+t}\Delta_p \gamma\right)\,,$$

$$p_\alpha C(Y) := i\frac{\partial}{\partial y^\alpha} C(Y) \qquad t_\alpha \omega(Y) := i\frac{\partial}{\partial y^\alpha}\omega(Y)$$

$\Upsilon(\omega,\omega,C)$ – vertex

$$\mathrm{d}_x\omega+\omega*\omega=\frac{\eta}{4i}(\omega*\omega*C*X_{\omega\omega C}+C*\omega*\omega*X_{C\omega\omega}+\omega*C*\omega*X_{\omega C\omega}),$$

$\Upsilon(\omega, \omega, C)$ – vertex

$$\mathrm{d}_x\omega + \omega * \omega = \frac{\eta}{4i}(\omega * \omega * C * X_{\omega\omega C} + C * \omega * \omega * X_{C\omega\omega} + \omega * C * \omega * X_{\omega C\omega}),$$

where

$$X_{\omega\omega C} = h_{p+t_1+t_2}\Delta_p\Delta_{p+t_2}\gamma\,,$$
$$X_{C\omega\omega} = h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+2t_1+2t_2}\gamma\,,$$
$$X_{\omega C\omega} = -h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+t_2}\gamma - h_{p+t_1+2t_2}\Delta_{p+2t_2}\Delta_{p+t_2}\gamma\,.$$

$\Upsilon(\omega, \omega, C)$ – vertex

$$\mathrm{d}_x\omega+\omega*\omega = \frac{\eta}{4i}(\omega*\omega*C*X_{\omega\omega C}+C*\omega*\omega*X_{C\omega\omega}+\omega*C*\omega*X_{\omega C\omega})\,,$$

where

$$X_{\omega\omega C} = h_{p+t_1+t_2}\Delta_p\Delta_{p+t_2}\gamma\,,$$
$$X_{C\omega\omega} = h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+2t_1+2t_2}\gamma\,,$$
$$X_{\omega C\omega} = -h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+t_2}\gamma - h_{p+t_1+2t_2}\Delta_{p+2t_2}\Delta_{p+t_2}\gamma\,.$$

explicitly

$$\Upsilon_{\omega\omega C} = \frac{\eta}{2i}\int_{[0,1]^3} d^3\tau\delta(1-\tau_1-\tau_2-\tau_3)e^{i(1-\tau_3)\partial_1^\alpha\partial_{2\alpha}}$$
$$\partial^\alpha\omega((1-\tau_1)y)\,\bar{*}\,\partial_\alpha\omega(\tau_2 y)\,\bar{*}\,C(-i\tau_1\partial_1-i(1-\tau_2)\partial_2)\,,$$

$\Upsilon(\omega, \omega, C)$ – vertex

$$\mathrm{d}_x\omega + \omega * \omega = \frac{\eta}{4i}(\omega * \omega * C * X_{\omega\omega C} + C * \omega * \omega * X_{C\omega\omega} + \omega * C * \omega * X_{\omega C\omega}),$$

where

$$X_{\omega\omega C} = h_{p+t_1+t_2}\Delta_p\Delta_{p+t_2}\gamma,$$
$$X_{C\omega\omega} = h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+2t_1+2t_2}\gamma,$$
$$X_{\omega C\omega} = -h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+t_2}\gamma - h_{p+t_1+2t_2}\Delta_{p+2t_2}\Delta_{p+t_2}\gamma.$$

explicitly

$$\Upsilon_{\omega\omega C} = \frac{\eta}{2i}\int_{[0,1]^3} d^3\tau\delta(1-\tau_1-\tau_2-\tau_3)e^{i(1-\tau_3)\partial_1^\alpha\partial_{2\alpha}}$$
$$\partial^\alpha\omega((1-\tau_1)y)\,\bar{*}\,\partial_\alpha\omega(\tau_2 y)\,\bar{*}\,C(-i\tau_1\partial_1 - i(1-\tau_2)\partial_2),$$

No y – dependence in $C(y)$ $\Rightarrow$ ultra local form

$\Upsilon(\omega, C, C)$ – vertex

Solving for $B(z, y)$ to the second order

$$B_2 := B_2^q = \frac{\eta}{4i} C * C * \Delta_q(\Delta_{p_2} - \Delta_{p_1+2p_2})\gamma\,, \quad q = v_1 p_1 + v_2 p_2$$

$\Upsilon(\omega, C, C)$ – vertex

Solving for $B(z,y)$ to the second order

$$B_2 := B_2^q = \frac{\eta}{4i} C * C * \Delta_q(\Delta_{p_2} - \Delta_{p_1+2p_2})\gamma\,, \quad q = v_1 p_1 + v_2 p_2$$

Locality requirement

$$v_2 - v_1 = 1$$

$\Upsilon(\omega, C, C)$ – vertex

Solving for $B(z, y)$ to the second order

$$B_2 := B_2^q = \frac{\eta}{4i} C * C * \Delta_q (\Delta_{p_2} - \Delta_{p_1+2p_2})\gamma\,, \qquad q = v_1 p_1 + v_2 p_2$$

Locality requirement

$$v_2 - v_1 = 1$$

$$\mathrm{d}_x C + [\omega, C]_* = \frac{\eta}{4i}(\omega{*}C{*}C{*}X_{\omega CC} + C{*}C{*}\omega{*}X_{CC\omega} + C{*}\omega{*}C{*}X_{C\omega C})$$

$\Upsilon(\omega, C, C)$ – vertex

Solving for $B(z, y)$ to the second order

$$B_2 := B_2^q = \frac{\eta}{4i} C * C * \Delta_q(\Delta_{p_2} - \Delta_{p_1+2p_2})\gamma\,, \qquad q = v_1 p_1 + v_2 p_2$$

Locality requirement

$$v_2 - v_1 = 1$$

$$\mathrm{d}_x C + [\omega, C]_* = \frac{\eta}{4i}(\omega{*}C{*}C{*}X_{\omega CC} + C{*}C{*}\omega{*}X_{CC\omega} + C{*}\omega{*}C{*}X_{C\omega C})$$

$$X_{\omega CC} = h_{p_2}\Delta_{p_1+2p_2}\Delta_{p_1+2p_2+t}\gamma\,,$$
$$X_{CC\omega} = h_{p_2+2t}\Delta_{p_2+t}\Delta_{p_1+2p_2+2t}\gamma\,,$$
$$X_{C\omega C} = (h_{p_1+2p_2+2t} - h_{p_2})\Delta_{p_2+t}\Delta_{p_1+2p_2+t}\gamma$$

$\Upsilon(\omega, C, C)$ – vertex

Explicitly,

$$\Upsilon_{\omega CC} = \frac{\eta}{2i} \int_{[0,1]^3} d^3\tau \delta(1-\tau_1-\tau_2-\tau_3)(\partial_1^\alpha + \partial_2^\alpha)\partial_\alpha^\omega$$
$$\omega((1-\tau_3)y)\, C(\tau_1 y - i(1-\tau_2)\partial^\omega)\, C(-(1-\tau_1)y + i\tau_2\partial^\omega)\,,$$

The result is perfectly local in accordance with PLT

Uniform homotopy shifts

Taking different homotopies $\alpha\neq 0$

$$S_1=-\frac{\eta}{2}\Delta_{\alpha(p+y)}(C*\gamma)+c.c.=-\frac{\eta}{2}C*\Delta_{p+\alpha y}\gamma\,,$$

$$W_1=\frac{1}{2i}\Delta_{\alpha(p+y)}(\mathrm{d}_x S_1+\omega*S_1+S_1*\omega)$$

$$=-\frac{\eta}{4i}\left(C*\omega*\Delta_{p+t+\alpha y}\Delta_{p+2t+\alpha y}\gamma-\omega*C*\Delta_{p+t+\alpha y}\Delta_{p+\alpha y}\gamma\right)\,.$$

Uniform homotopy shifts

Taking different homotopies $\alpha \neq 0$

$$S_1 = -\frac{\eta}{2}\Delta_{\alpha(p+y)}(C * \gamma) + c.c. = -\frac{\eta}{2} C * \Delta_{p+\alpha y}\gamma\,,$$

$$W_1 = \frac{1}{2i}\Delta_{\alpha(p+y)}(\mathrm{d}_x S_1 + \omega * S_1 + S_1 * \omega)$$

$$= -\frac{\eta}{4i}\left(C * \omega * \Delta_{p+t+\alpha y}\Delta_{p+2t+\alpha y}\gamma - \omega * C * \Delta_{p+t+\alpha y}\Delta_{p+\alpha y}\gamma\right)\,.$$

Analogously,

$$B_2 = \frac{\eta}{4i} C * C * \Delta_{p_1+2p_2+\alpha y}\Delta_{p_2+\alpha y}\gamma$$

Uniform homotopy shifts

Taking different homotopies $\alpha \neq 0$

$$S_1 = -\frac{\eta}{2}\Delta_{\alpha(p+y)}(C*\gamma) + c.c. = -\frac{\eta}{2} C*\Delta_{p+\alpha y}\gamma\,,$$

$$W_1 = \frac{1}{2i}\Delta_{\alpha(p+y)}(\mathrm{d}_x S_1 + \omega * S_1 + S_1 * \omega)$$

$$= -\frac{\eta}{4i}\left(C*\omega*\Delta_{p+t+\alpha y}\Delta_{p+2t+\alpha y}\gamma - \omega * C * \Delta_{p+t+\alpha y}\Delta_{p+\alpha y}\gamma\right)\,.$$

Analogously,

$$B_2 = \frac{\eta}{4i} C*C*\Delta_{p_1+2p_2+\alpha y}\Delta_{p_2+\alpha y}\gamma$$

Result in identically equivalent vertices $\Upsilon(\omega,\omega,C)$ and $\Upsilon(\omega,C,C)$ as for $\alpha = 0$

Conclusion

- One of the major and simple results of our study are the star-product exchange formulas that relate shifted homotopies and the the star product.

Conclusion

- One of the major and simple results of our study are the star-product exchange formulas that relate shifted homotopies and the the star product.
- Simplest quartic vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, C, C)$ were shown to have ultra local and local form correspondingly.

Conclusion

- One of the major and simple results of our study are the star-product exchange formulas that relate shifted homotopies and the the star product.
- Simplest quartic vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, C, C)$ were shown to have ultra local and local form correspondingly.
- A hidden symmetry realized as certain uniform homotopy shifts that respects the obtained vertices was observed.