Probability of radiation of twisted photons in the infrared domain

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Outline

1. Twisted photons: generation, applications, observations.
2. Problem statement and results.
3. Probability of radiation of twisted photons by classical currents.
4. IR asymptotics of radiation of twisted photons. The edge radiation:
   - Trajectories with a break on the detector axis.
   - Trajectories with a break out of the detector axis.
5. Conclusion.

Definition

The twisted photons are the states of a free electromagnetic field with the definite energy k_0, the longitudinal projection of momentum k_3, the projection of the total angular momentum m, and the helicity s.

Generation

1. Spatial light modulators and digital micro-mirror devices (optical range and below).
2. Spiral phase plates and diffraction gratings with ℓ-pronged forks dislocations (x-ray range and below).
3. Helical undulators (x-ray range and below).
4. Inverse Compton scattering of low energy twisted photons and channeling (hundreds of MeVs and below).

Applications

1. High-density information transfer.
2. Manipulation of the rotational degrees of freedom of irradiated objects: microstructures, molecules, atoms, nuclei, hadrons.
3. High-contrast microscopy.
4. Particle trapping and optical tweezers.

Observation of twisted photons at a single-photon level

1. Dove-prism interferometers.
2. Spatial light modulator based devices: holograms, optical transformation approaches.

Problem we are going to address

- The IR asymptotics of radiation of photons is universal for any QED process [F. Bloch, A. Nordsieck, Phys. Rev., (1937) and, e.g., S. Weinberg, Phys. Rev., (1965)].
- This radiation is called the edge radiation. It corresponds to the radiation produced by the classical current of free charged particles in the in- and out-states. This radiation is well studied in terms of the plane-wave photons.
- We study the properties of the edge radiation in terms of twisted photons.

Results

1. The exact formula for the radiation probability of twisted photons in the far infrared is obtained.
2. The symmetry property of the average number of twisted photons produced in any QED process in the far infrared is established.
3. The main characteristics of the radiation twist – the differential asymmetry, the average projection of the total angular momentum of radiation, and the projection of the total angular momentum per photon – are found.

Results (continuation)

4. In the ultrarelativistic limit, the probability of radiation of twisted photons appears to have a sharp peak near $n_\perp \approx \beta_\perp$, where $n_\perp = k_\perp/k_0$ and $k_\perp := \sqrt{k_0^2 - k_3^2}$.
5. The maximum value of total angular momentum per photon is given by

$$|\ell_{max}| \approx k_\perp |x_\perp| \Big[1 - K^{-1}\Big(\frac{3}{2}\Big)^{3/2}\Big] + \frac{K}{2\sqrt{2}}, \qquad (1)$$

$K := \beta_\perp \gamma$,

$|x_\perp|$ is the shortest distance from the detector axis to the trajectory of a charged particle.

6. Several section rules are found:
   - If the current density $j^i(t, \mathbf{x})$ is invariant under the rotation by an angle of $2\pi/r$, $r \in \mathbb{N}$, around the detector axis for all t, then

$$dP(s, m, k_3, k_\perp) = 0, \qquad m \neq lr, \quad l \in \mathbb{Z}. \qquad (2)$$

Results (continuation)

- If the trajectories of identical charged particles are obtained from one trajectory by the rotation by an angle of φ_k, the translation along the detector axis by x_3^k, and the translation in time x_0^k, where

$$\varphi_k = \frac{2\pi k}{r}, \qquad x_3^k = \frac{\lambda_0}{2\pi}\varphi_k, \qquad x_0^k = \frac{\lambda_0}{2\pi\beta_\parallel}\varphi_k, \qquad k = \overline{1, r}, \tag{3}$$

  and λ_0 and $\beta_\parallel$ are some fixed parameters, then there is the selection rule

$$m = \operatorname{sgn}(\lambda_0)n + lr, \qquad k_0 = k_0^n := \frac{2\pi n}{|\lambda_0|(\beta_\parallel^{-1} - n_3)}, \qquad n \in \mathbb{N},\ l \in \mathbb{Z}, \tag{4}$$

  $n_3 := k_3/k_0$, at the sharp maxima of radiation probability of twisted photons.
- If the initial and final velocities of a charged particle and the detector axis lie in one plane, the initial and final energies and the projections of the initial and final velocities to the detector axis are the same, then m is an odd number in the far infrared.

Basis

$$\mathbf{e}_\pm := \mathbf{e}_1 \pm i\mathbf{e}_2. \tag{5}$$

$\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is a right-handed orthonormal triple.
$\mathbf{e}_3$ is directed along the the detector axis.

Vector decomposition

$$\mathbf{x} = \frac{1}{2}(x_-\mathbf{e}_+ + x_+\mathbf{e}_-) + x_3\mathbf{e}_3. \tag{6}$$

$x_\pm = (\mathbf{x}, \mathbf{e}_\pm)$ and $x_3 = (\mathbf{x}, \mathbf{e}_3)$.

Process of radiation of photons

$$0 \to \gamma_\alpha + X. \tag{7}$$

0 is the vacuum state.
γ_α is the photon in the state α recorded by the detector.
X is for the other created photons.

Probability of the inclusive process (7)

$$w_{incl}(\alpha; 0) = 1 - e^{-n(\alpha;0)} \approx n(\alpha; 0). \tag{8}$$

$n(\alpha; 0)$ is the average number of photons in the state α created by the current $j_\mu(x)$.

Radiation of twisted photons by classical currents

Density of the average number of twisted photons created by the current of N particles
[O.V. Bogdanov, *et al.*, Phys. Rev. A **97**, 033837 (2018)]

$$dP(s,m,k_3,k_\perp) = \Big| \sum_{l=1}^{N} e_l \int d\tau_l e^{-i[k_0 x_l^0(\tau_l) - k_3 x_{l3}(\tau_l)]} \Big\{ \frac{1}{2} \big[\dot{x}_{l+}(\tau_l) a_-(s,m,k_3,k_\perp;\mathbf{x}_l(\tau_l)) + \dot{x}_{l-}(\tau_l) a_+(s,m,k_3,k_\perp;\mathbf{x}_l(\tau_l)) \big] + \dot{x}_{l3}(\tau_l) a_3(m,k_\perp;\mathbf{x}_l(\tau_l)) \Big\} \Big|^2 \Big(\frac{k_\perp}{2k_0} \Big)^3 \frac{dk_3 dk_\perp}{2\pi^2}. \quad (9)$$

k_0 is the photon energy. $k_\perp := \sqrt{k_0^2 - k_3^2}$.
$s = \pm 1$ is the photon helicity.
$m \in \mathbb{Z}$ is the projection of the total angular momentum of a photon onto the detector axis.
The origin of the reference frame is taken on the line passing through the detector axis.

Mode functions

[e.g., R. Jáuregui, *et al.*, Phys. Rev. A **71**, 033411 (2005)]

$$\begin{aligned} a_3(m,k_\perp;\mathbf{x}) &= \frac{x_+^{m/2}}{x_-^{m/2}} J_m(k_\perp x_+^{1/2} x_-^{1/2}) =: j_m(k_\perp x_+, k_\perp x_-), \\ a_\pm(s,m,k_3,k_\perp;\mathbf{x}) &= \frac{ik_\perp}{sk_0 \pm k_3} j_{m\pm1}(k_\perp x_+, k_\perp x_-). \end{aligned} \quad (10)$$

Infrared asymptotics of radiation of twisted photons

Infrared asymptotics corresponds to the trajectory of a charged particle

$$\mathbf{x}_l(t) = \begin{cases} \mathbf{x}_{0l} + \mathbf{v}_l t, & t > 0; \\ \mathbf{y}_{0l} + \mathbf{u}_l t, & t < 0. \end{cases} \tag{11}$$

$\mathbf{x}_{0l}$ and $\mathbf{v}_l$ specify the future asymptote of the trajectory of lth particle.
$\mathbf{y}_{0l}$ and $\mathbf{u}_l$ specify the past asymptote of the trajectory of lth particle.

Applicability conditions for $\gamma \gtrsim 10$

$$k_3 |\delta x_{l3}(\omega t)|_{\omega t \gtrsim 2\pi} \ll 1, \qquad k_\perp |\delta x_{l+}(\omega t)|_{\omega t \gtrsim 2\pi} \ll 1,$$
$$2\pi k_0 \lesssim \frac{2\omega\gamma^2}{1 + K^2(1 + n_k^2)}. \tag{12}$$

$K := \beta_\perp \gamma$ and $n_k := n_\perp / \beta_\perp$.
$\delta x_{l3}(\omega t)$ and $\delta x_{l+}(\omega t)$ are the deviations of the trajectory from the asymptote (11).
ω^{-1} characterizes the time when the particles move with acceleration.

Warning

Since the probability of radiation of twisted photons is not invariant under the translations of the origin that are perpendicular to the detector axis, it is relevant where the break of the trajectory is located.

Trajectories with a break on the detector axis

Contribution to the radiation amplitude from the future asymptote

$$I_3+\frac{1}{2}(I_++I_-)=\frac{i^{-1-m}}{k_0n_\perp^2}e^{im\delta}\Big(\frac{v_3-n_3}{\kappa(v)}-s\,\mathrm{sgn}(m)\Big)q^{|m|}(v),\qquad \text{for } |m|>1;$$

$$I_3+\frac{1}{2}(I_++I_-)=\frac{i^{-1}}{k_0n_\perp^2}\Big(\frac{v_3-n_3}{\kappa(v)}+n_3\Big),\qquad \text{for } m=0; \tag{13}$$

$$\kappa(v):=\big[(1-n_3v_3)^2-n_\perp^2\beta_\perp^2\big]^{1/2},$$
$$q(v):=\frac{n_\perp\beta_\perp}{1-n_3v_3+\kappa(v)}=\frac{1-n_3v_3-\kappa(v)}{n_\perp\beta_\perp},\qquad 0\leq q<1. \tag{14}$$

$\delta:=\arg v_+$.

Symmetry property

$$dP(s,m,k_3,k_\perp)=dP(-s,-m,k_3,k_\perp). \tag{15}$$

Reflection symmetry

The symmetry property (15) holds for any process of radiation of twisted photons in the far infrared.

Trajectories with a break on the detector axis

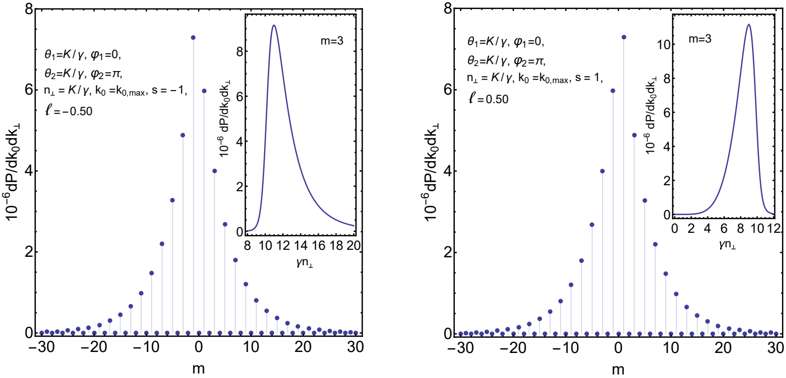

Figure: The elastic scattering of the electron with the Lorentz-factor $\gamma = 10^3$ off the target positioned on the detector axis. The photon energy $k_0 = 87$ eV. The parameter $K := \beta_\perp \gamma = 10$. The angles θ_1, φ_1 are the polar and azimuth angles of the initial velocity of the electron with respect to the detector axis. The angles θ_2, φ_2 are the polar and azimuth angles of the final velocity of the electron with respect to the detector axis. The distributions over m obey the symmetry property (15). The insets: The dependence of the average number of twisted photons on $n_\perp$ at $m = 3$ and the helicities $s = \pm 1$.

Trajectories with a break on the detector axis

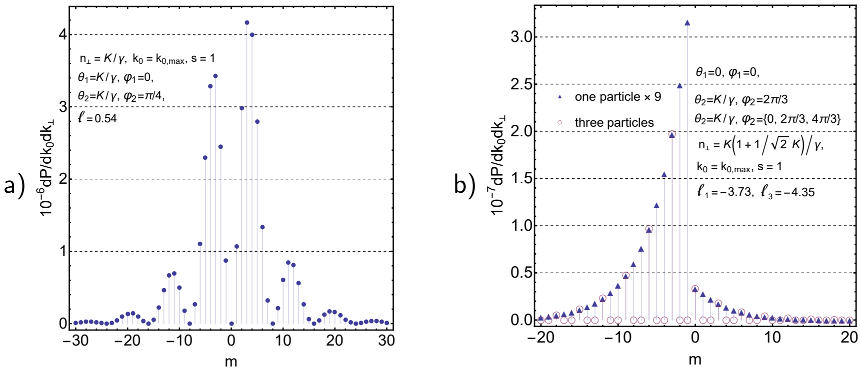

Figure: The elastic scattering of the electron off the target positioned on the detector axis for the different initial and final directions of the electron velocity. (a) The minima of the average number of twisted photons occur with the period $T_m = 2\pi/|\delta_{12}|$, where $\delta_{12} = \arg v_+ - \arg u_+$. (b) The production of twisted photons in scattering of one and three electrons is compared. The distributions over m satisfy the selection rule (2) for symmetric currents.

Trajectories with a break out of the detector axis

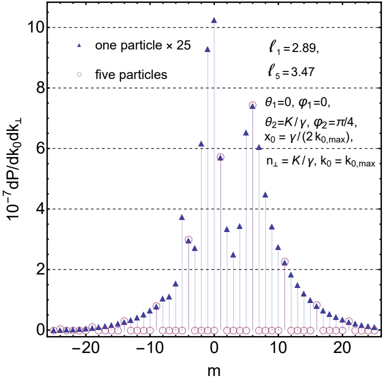

Figure: The average number of twisted photons summed over helicities. The target is positioned at $(x_0, 0, 0)$, where $x_0 \approx 1.1$ μm. The production of twisted photons in scattering of one and five electrons is compared. The trajectories of five electrons are obtained from one electron trajectory by the rotation around the detector axis, the translation along it, and the translation in time (3). This system models the scattering of the bunch of charged particles on the spiral staircase made of crystals. The parameter $\lambda_0 \approx 283$ μm is taken from (4) such that k_0 is the first harmonic, and $\beta_{||} = (1 - 1/\gamma^2)^{1/2}$. The fulfillment of the selection rule (4) is clearly seen.

Trajectories with a break out of the detector axis

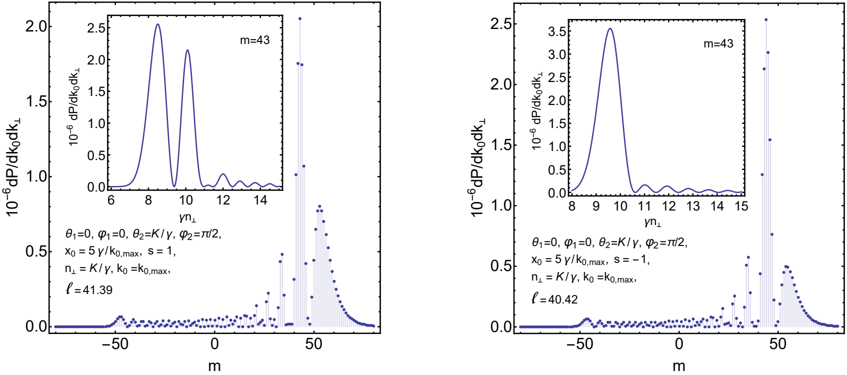

Figure: The parameter $x_0 \approx 11$ μm and $k_\perp |x_+| = 50$. The local minimum is near $m = -k_\perp x_\perp = 50$. The insets: The dependence of the average number of twisted photons on $n_\perp$ at $m = 43$ and the helicities $s = \pm 1$.

Conclusion (see for more arXiv:1803.03447)

Synopsis of the results

1. The IR asymptotics of the radiation of twisted photons for QED processes is investigated.
2. This asymptotics is universal and can be described by the radiation of the classical current corresponding to charged particles moving uniformly along straight lines with the break at the instant of time $t = 0$ (the edge radiation).
3. We show that this radiation can produce twisted photons with large angular momentum.
4. As the edge radiation is infrared, it can be used as a superradiant coherent source of twisted photons with large angular momentum.