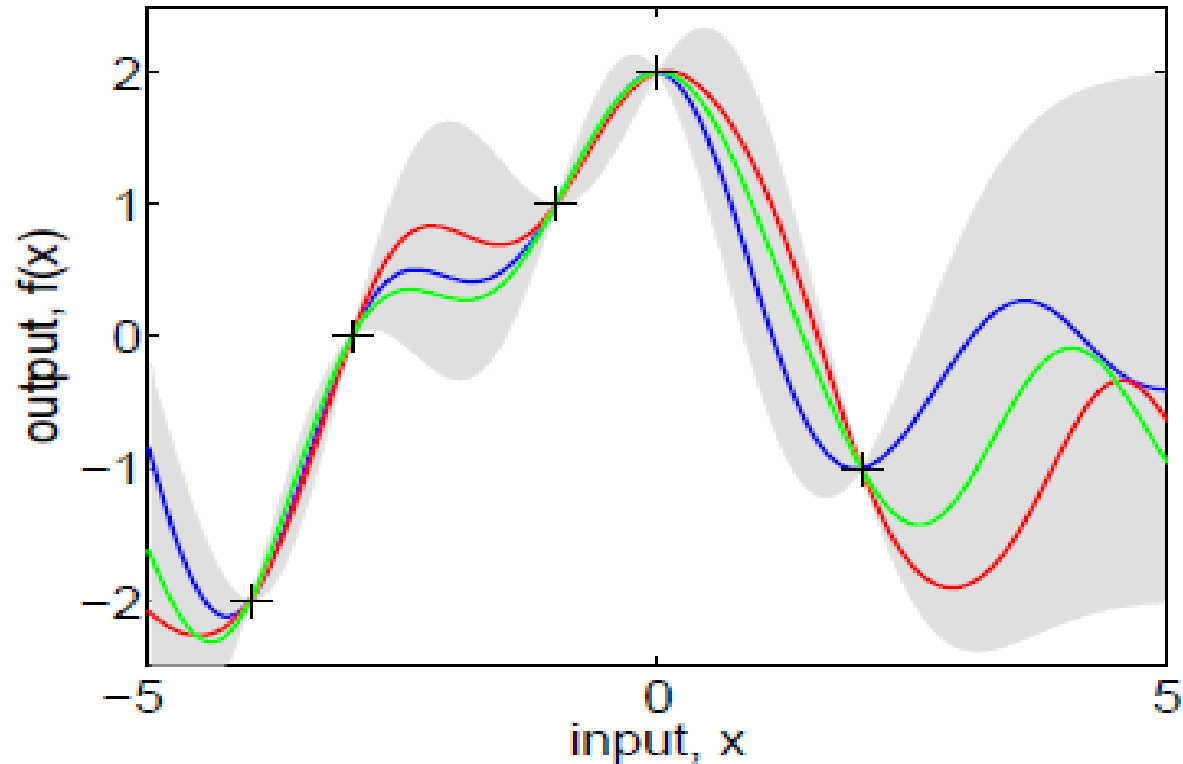


# Interacting dark energy models in light of Gaussian processes



Martiros Khurshudyan

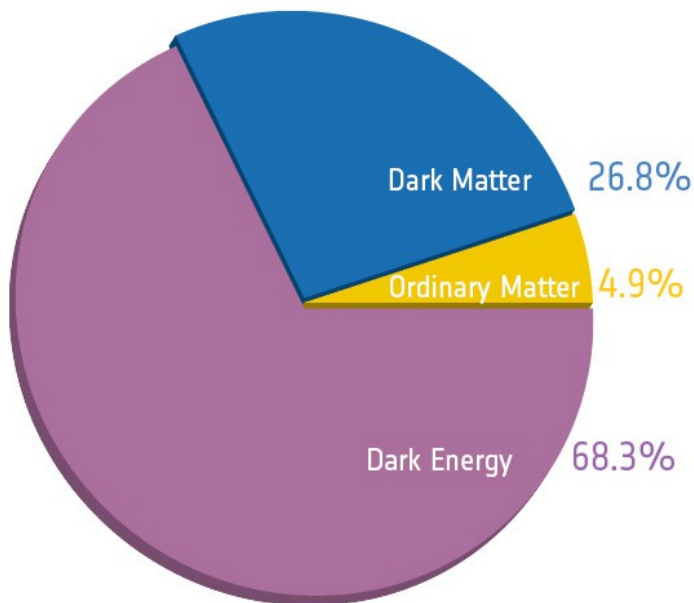
University of Science and Technology of China

# Interacting dark energy models in light of Gaussian processes

- Short introduction – known facts
- Motivation and specific DE model
- Gaussian Processes and  $H(z)$  data
- Interacting Dark Energy models and GP
- Some results
- Conclusion

# Short introduction – known facts

## Large scale universe



**Dark energy** has enough negative pressure to work against gravity.

**Models of dark energy** – quintessence, phantom, k-essence, ghost dark energy, holographic dark energy, Chaplygin gas ...

## Alternative approach

a modification of gravity

Planck Collaboration, arXiv:1502.01589 (2015)

J. Yoo, Y. Watanabe, Int. J. Mod. Phys. D 21, 1230002 (2012)

S. Nojiri, S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115-146 (2007)

T. Clifton et al., Physics Reports 513, 1-189 (2012)

# Motivation and specific DE model

## Motivation of the research

The result reported by the BOSS experiment for the Hubble parameter at  $z = 2.34$  is a direct evidence for the existence of a non-gravitational coupling between dark energy and dark matter.

$$H(2.34) = 222 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

## Our dark energy model

$$\omega_{\text{de}} = \omega_0 + \omega_1 q$$

where  $\omega_0$  and  $\omega_1$  are parameters of the model

$q$  is the deceleration parameter

$$q(z) = -1 + \frac{1+z}{H} H'$$

## The main results obtained from direct numerical integration

$H(2.34) = 222 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$  result can be explained without interaction  
and  
the EoS we have considered is in no way strange

**But this is fully model dependent result!!!!**

# Gaussian processes and H(z) data

*The Gaussian distribution presents a distribution of a random variable characterized by a mean and a covariance.*

*GP should be understood as a distribution over functions, characterized by a mean function and a covariance matrix.*

The covariance function (kernel) which correlates the function  $H(z)$  at different points

$$k(z, \hat{z}) = \sigma_f^2 \exp \left[ -\frac{|z - \hat{z}|^2}{2l^2} \right]$$

where  $\sigma_f$  and  $l$  are the parameters known as hyperparameters. These parameters represent the length scales in the GP.  $l$  parameter corresponds to the correlation length along which the successive  $f(x)$  values are correlated, while to control the variation in  $f(x)$  relative to the mean of the process we need  $\sigma_f$  parameter.

$z$	$H(z)$	$\sigma_H$	$z$	$H(z)$	$\sigma_H$
0.070	69	19.6	0.4783	80.9	9
0.090	69	12	0.480	97	62
0.120	68.6	26.2	0.593	104	13
0.170	83	8	0.680	92	8
0.179	75	4	0.781	105	12
0.199	75	5	0.875	125	17
0.200	72.9	29.6	0.880	90	40
0.270	77	14	0.900	117	23
0.280	88.8	36.6	1.037	154	20
0.352	83	14	1.300	168	17
0.3802	83	13.5	1.363	160	33.6
0.400	95	17	1.4307	177	18
0.4004	77	10.2	1.530	140	14
0.4247	87.1	11.1	1.750	202	40
0.44497	92.8	12.9	1.965	186.5	50.4
0.24	79.69	2.65	0.60	87.9	6.1
0.35	84.4	7	0.73	97.3	7.0
0.43	86.45	3.68	2.30	224	8
0.44	82.6	7.8	2.34	222	7
0.57	92.4	4.5	2.36	226	8

TABLE I: The  $H(z)$  and its uncertainty  $\sigma_H$  are in the unit of  $\text{km s}^{-1} \text{Mpc}^{-1}$ .

***We use GaPP code by Marina Seikel et al...***

# Gaussian processes

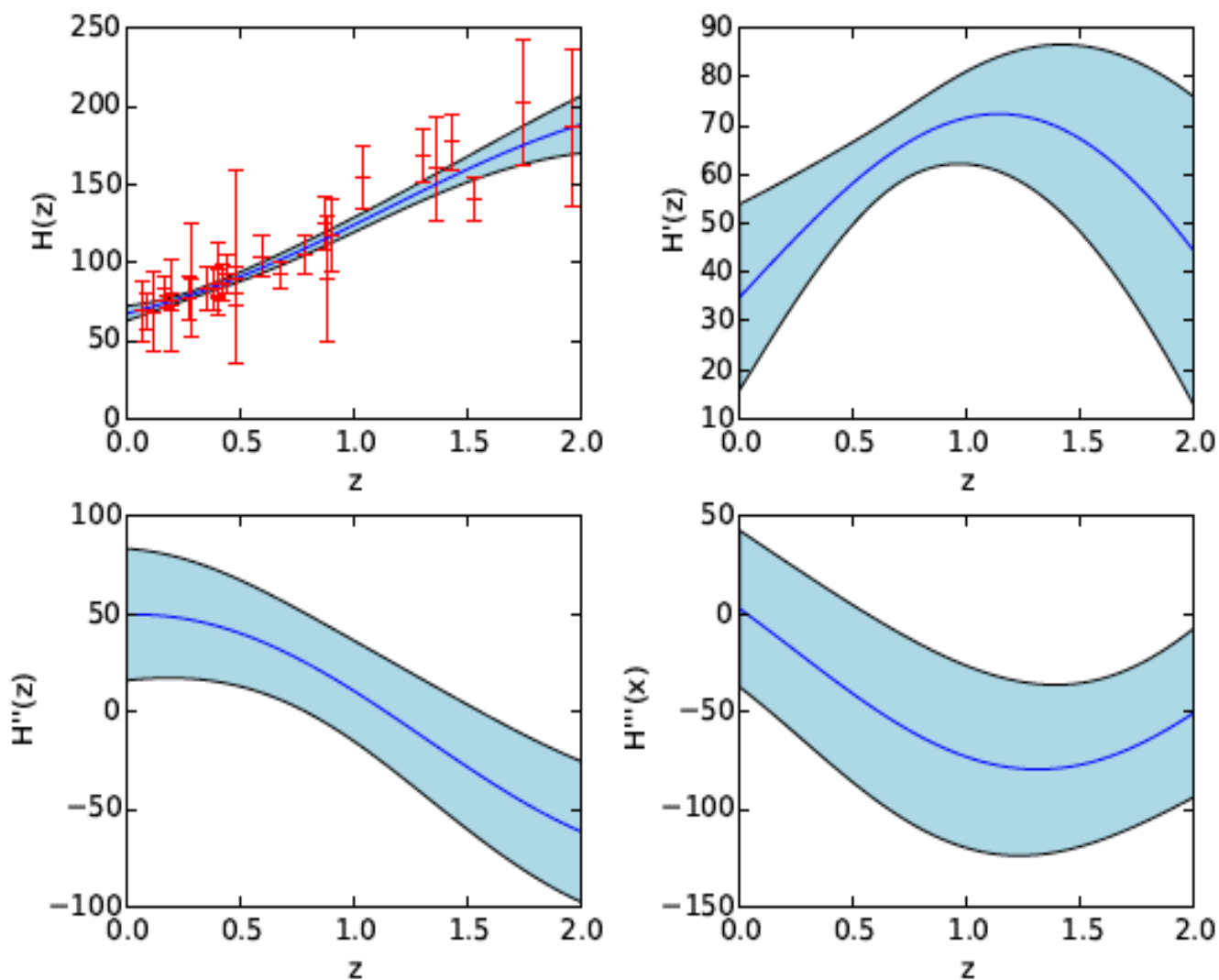


FIG. 1: GP reconstruction of  $H(z)$ ,  $H'(z)$ ,  $H(z)''$ , and  $H(z)'''$  for the 30-point sample deduced from the differential age method. The ' means derivative with respect to the redshift  $z$ .

# Gaussian processes

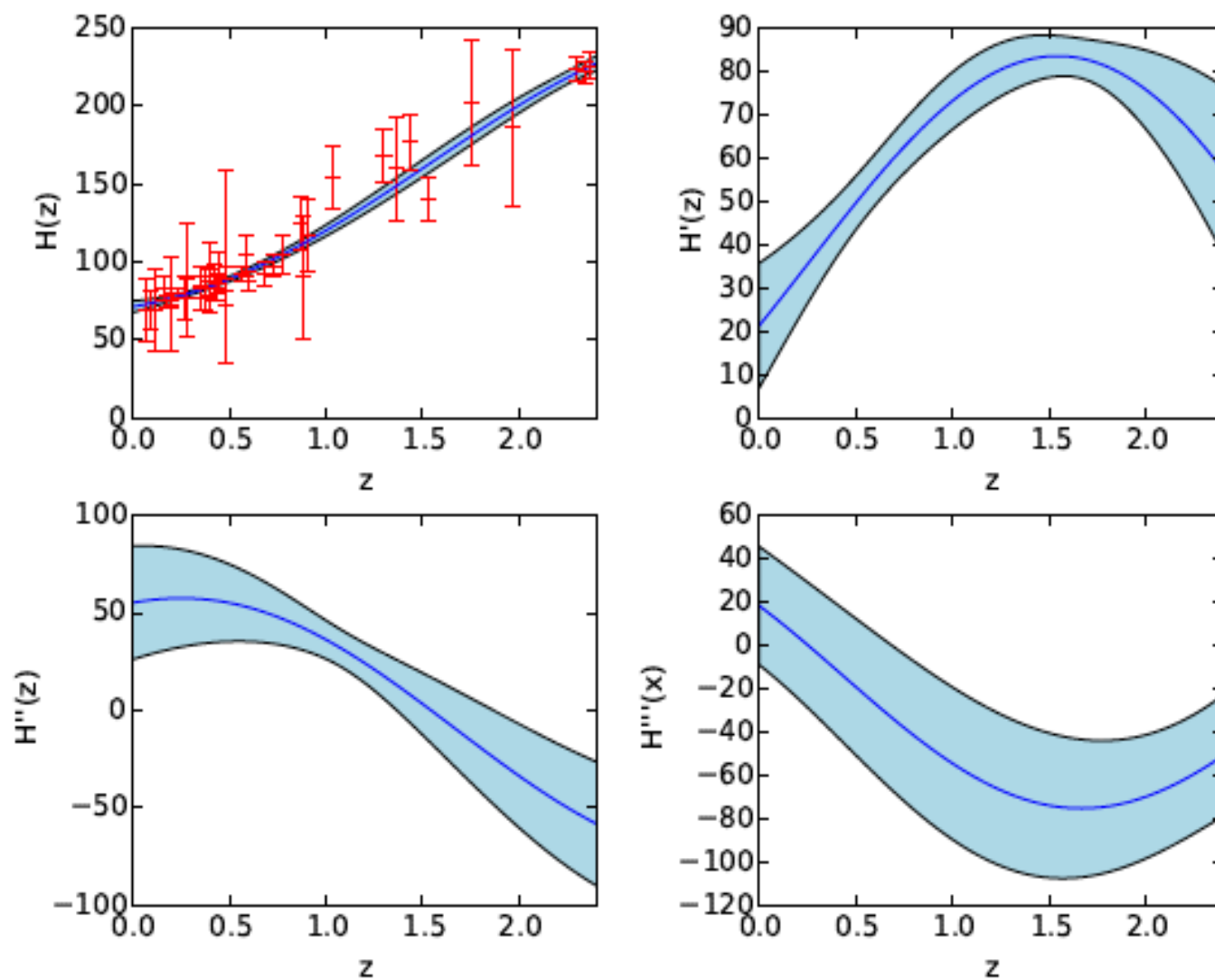


FIG. 2: GP reconstruction of  $H(z)$ ,  $H'(z)$ ,  $H''(z)$ , and  $H'''(z)$  for the 30-point sample deduced from the differential age method, with the additional 10-point sample obtained from the radial BAO method. The ' means derivative with respect to the redshift  $z$ .

# Interacting Dark Energy models and GP

How we can use GP with IDE?

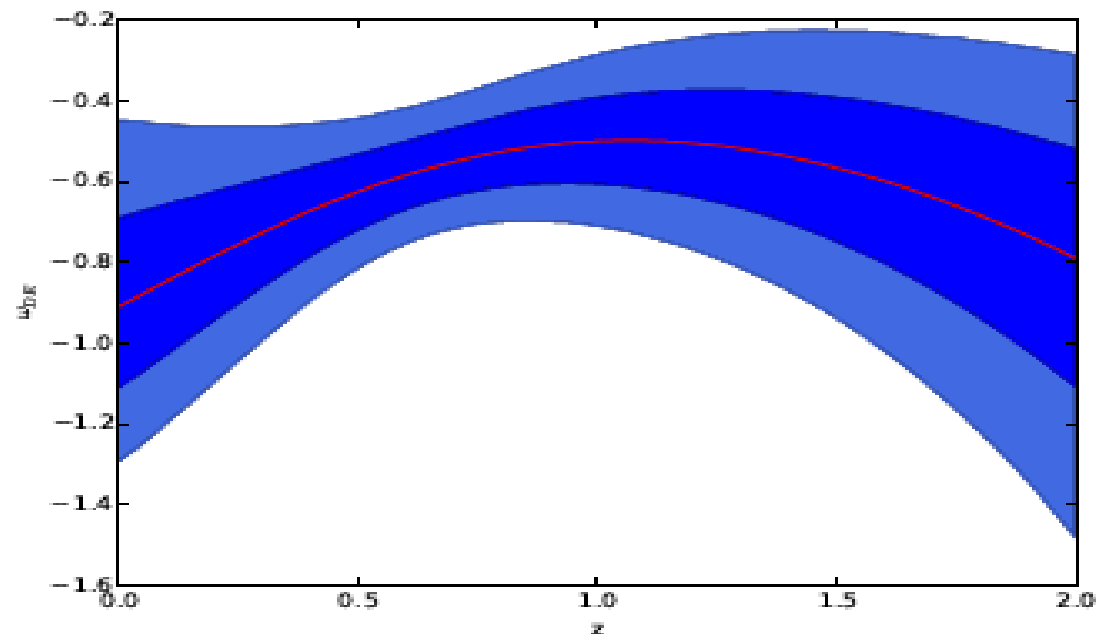
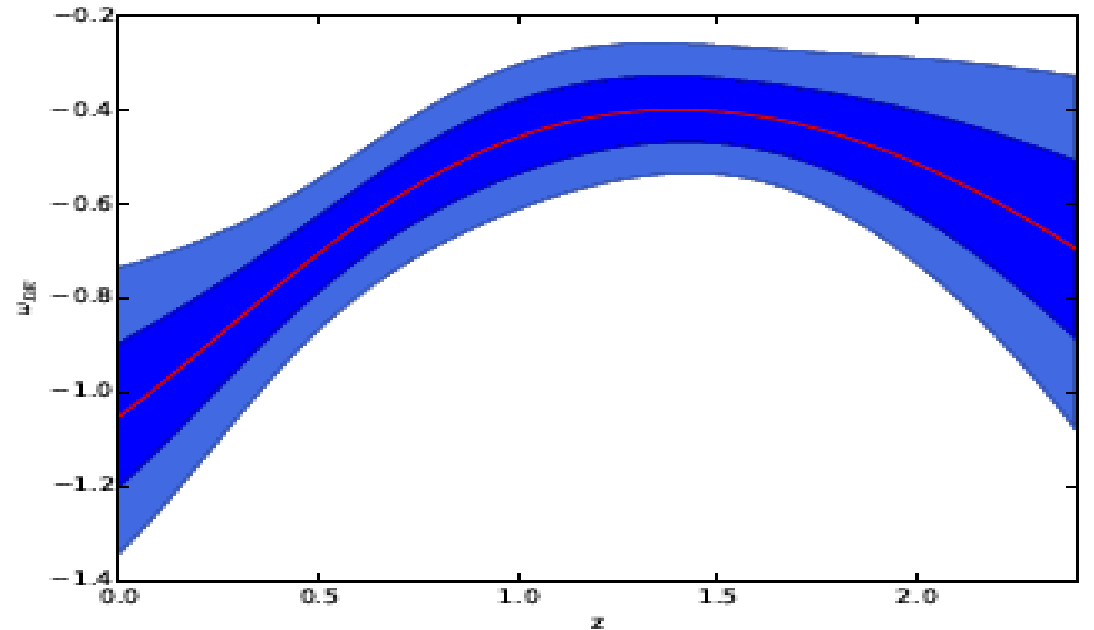
It is very simple task if EoS of DE is given

$$\dot{\rho}_{de} + 3H\rho_{de}(1 + \omega_{de}) = -Q$$

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$

$$H^2 = \frac{1}{3}\rho \quad \dot{H} = -\frac{1}{2}(\rho + 3P)$$

eventually the interaction term  $Q$  will be expressed as a function of EoS of DE, the Hubble parameter and its higher order derivatives





# Some results

1. The model according 68% C.L. of the reconstruction with 40 sample  $H(z)$  data can be accepted.
2. The model according 68% C.L. of the reconstruction with 30 sample  $H(z)$  data can be accepted.
3. According to that, either a

Type I (“The Big Rip Singularity”). If the singularity occurs at  $t = t_s$ , then the scale factor  $a$ , the effective energy density  $\rho_{\text{eff}}$ , and the pressure  $P_{\text{eff}}$ , diverge as  $t \rightarrow t_s$ ; that is,  $a \rightarrow \infty$ ,  $\rho_{\text{eff}} \rightarrow \infty$  and  $|P_{\text{eff}}| \rightarrow \infty$ . This case yields incomplete null and time-like geodesics,

Type III (“The Big Freeze Singularity”). In this case, only the scale factor is finite, and the effective pressure and effective density diverge at  $t \rightarrow t_s$ ; that is,  $a \rightarrow a_s < \infty$ ,  $\rho_{\text{eff}} \rightarrow \infty$  and  $|P_{\text{eff}}| \rightarrow \infty$ . These can be either weak or strong singularities, which are geodesically complete solutions.

or an  $\omega$  -singularity (Type V) can be generated

(with  $a \rightarrow \infty$ ,  $\rho_{\text{eff}} \rightarrow 0$  and  $|P_{\text{eff}}| \rightarrow 0$ , with EoS parameter  $\omega \rightarrow \infty$ , for  $t \rightarrow t_s$ )

# Some results

The phase space analysis with  $x = \frac{\rho_{\text{de}}}{3H^2}$  and  $y = \frac{P_{\text{de}}}{3H^2}$  gives

C.P.	$x$	$y$	$q$	$\omega_{\text{de}}$	$\omega_{\text{eff}}$	$\Omega_{\text{de}}/\Omega_{\text{dm}}$	Type of stability
C.P.1	$\frac{1}{b+1}$	-1	-1	$-b-1$	-1	$\frac{1}{b}$	Stable Node or Focus
C.P.2	$\frac{2(b+\omega_0)+\omega_1}{3b\omega_1+2\omega_0+\omega_1}$	$\frac{2(b+\omega_0)+\omega_1}{2-3\omega_1}$	$\frac{3b+3\omega_0+1}{2-3\omega_1}$	$\frac{3b\omega_1+2\omega_0+\omega_1}{2-3\omega_1}$	$\frac{2(b+\omega_0)+\omega_1}{2-3\omega_1}$	$\frac{2(b+\omega_0)+\omega_1}{b(3\omega_1-2)}$	Stable Node

TABLE II: Two stable critical points (C.P.) corresponding to the interacting model with  $Q = 3Hb\rho_{\text{de}}$ . The type of stability has been analyzed imposing the following prior constraints:  $\omega_0 \in [-1, 1]$ ,  $\omega_1 \in [-1, 1]$ , and  $b \geq 0$ .

C.P.	$x$	$y$	$q$	$\omega_{\text{de}}$	$\omega_{\text{eff}}$	$\Omega_{\text{de}}/\Omega_{\text{dm}}$	Type of stability
C.P.3	1	-1	-1	-1	-1	--	Stable Node
C.P.4	$\frac{-2b}{-3b\omega_1+2\omega_0+\omega_1}$	$-b$	$\frac{1-3b}{2}$	$\frac{-3b\omega_1+2\omega_0+\omega_1}{2}$	$-b$	$\frac{-2b}{b(2-3\omega_1)+2\omega_0+\omega_1}$	Stable Node
C.P.5	1	$\frac{2\omega_0+\omega_1}{2-3\omega_1}$	$\frac{1}{2} \left( \frac{3(2\omega_0+\omega_1)}{2-3\omega_1} + 1 \right)$	$\frac{2\omega_0+\omega_1}{2-3\omega_1}$	$\frac{2\omega_0+\omega_1}{2-3\omega_1}$	--	Stable Node

TABLE III: Two stable critical points corresponding to the interacting model with  $Q = 3Hb\rho_{\text{dm}}$ . The type of stability has been analysed by imposing the following prior constraints:  $\omega_0 \in [-1, 1]$ ,  $\omega_1 \in [-1, 1]$ , and  $b \geq 0$ .

# Conclusion

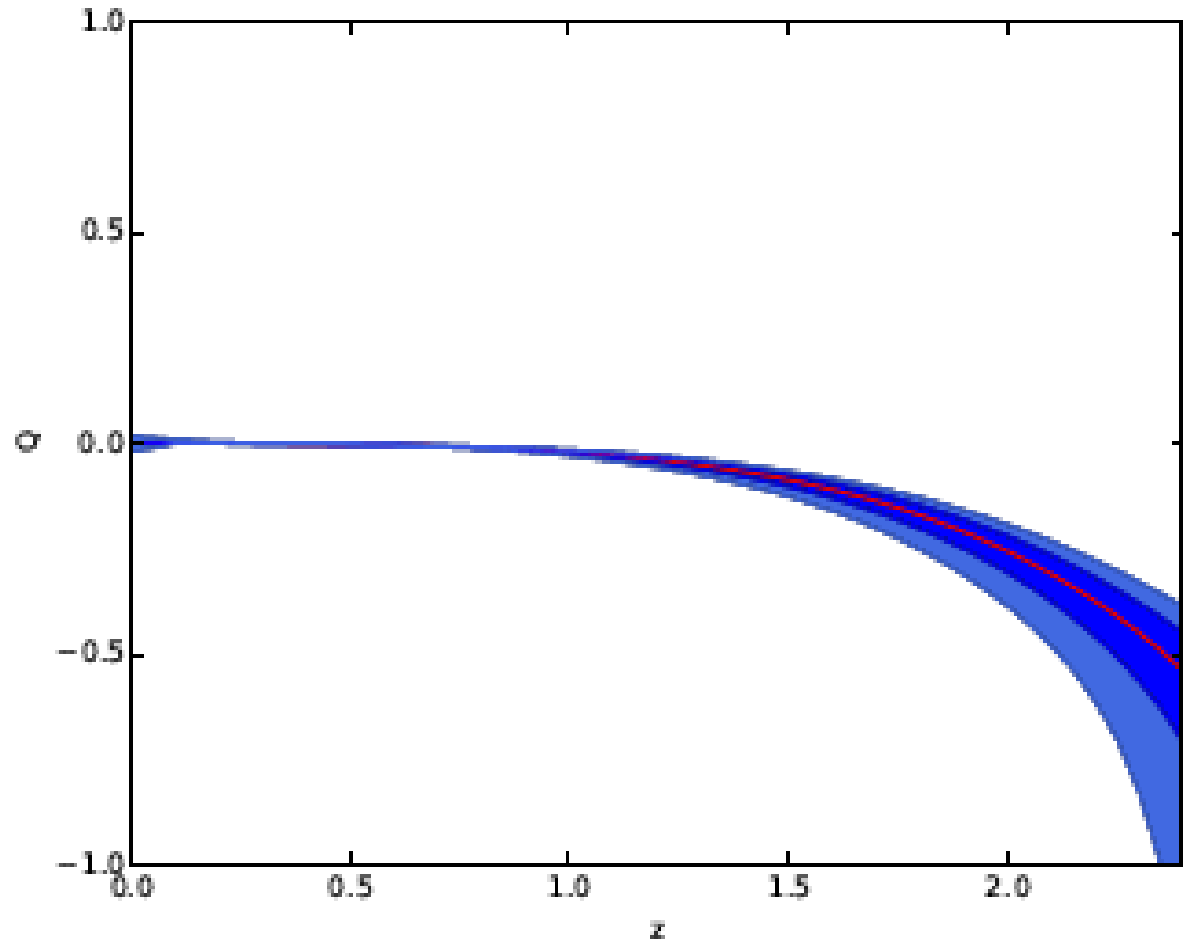
## The main message from direct numerical integration

1. The model can explain reported value of the Hubble parameter without non-gravitational interaction.
2. The EoS we have considered is in no way strange

## The main message from GP

1. we need in this case to involve a non-gravitational interaction between dark energy and dark matter, to explain the mentioned value of the Hubble parameter.
2. However, even if it would seem we do need to include a non-gravitational interaction, this occurs for redshifts not covered by recent  $H(z)$  data.

**we do not have a final answer to the main question!!!!**



**Thank You for Your attention**