Supersymmetric Higher Spin Interactions:

Conserved Supercurrents and Cubic Vertices

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- A collaboration with I. L. Buchbinder, S. J. Gates and R. von Unge

Why Study Higher Spins ?

- Fact: $0 \leq$ spin of elementary particles ≤ 2 $\leftarrow$ No-Go theorems

- First Motivation: Relativistic Field Theory in full generality

Particles
Unitary, irreps of
Poincaré Group $\longrightarrow$ Classify them (m,j) $\longrightarrow$ Physical requirements $(m^2 \geq 0,\, j < \infty)$

No reason to stop at $j \leq 2$

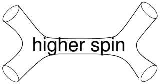
- Second Motivation: String Theory

$m = 0,\, j = 1$

$m = 0,\, j = 2$

$+$ tower of massive spins

$E \gg m \rightarrow$ {ignore massess} $\rightarrow$ {symmetries emerge} $\longrightarrow$ Higher Spin gauge symmetry



UV success $\longleftarrow$ Higher Spin propagation

Why Study Higher Spins ?

- Third Motivation: Holography (AdS/CFT)
  [QFT in fixed spacetime (D) = Gravity in different spacetime (AdS) (D+1)]

Succesful Models for
higher spin interactions
(Vasiliev's theory, 3D CS hs-gravity)

Two features
$m = 0,\, j = 2$
Not Flat Spacetime (AdS)

Higher spins a playground
for Holography

Interacting Theories

- Fact: Consistent Interacting Theories = Difficult Problem
  (without guiding principle)
- spin 1 (YM): Principle Bundles
- spin 2 (GR): Riemannian Manifolds
- higher spins: No Geometrical Input $\rightarrow$ Alternative methods
- Physical requirement: d.o.f. interacting theory = d.o.f. free theory

Gauge Invariance $\leftarrow$

Consistent higher spin interactions respect gauge symmetries

Noether's Method

Noether's method: A systematic, perturbative, analysis of invariance

Coupling Matter to Higher Spins

- Two types of fields: matter fields (ϕ), gauge fields (h)
- Expand the interacting action $S[\phi, h]$ in terms of coupling constant g
  the field transformations $\delta\phi$, δh

$$
\begin{aligned}
S[\phi,h] &= S_0[\phi] + g\ S_1[\phi,h] + g^2\ S_2[\phi,h] + \dots\ , \\
\delta\phi &= 0 + g\ \delta_1\phi + g^2\ \delta_2\phi + \dots\ , \\
\delta h &= \delta_0 h + g\ \delta_1 h + g^2\ \delta_2 h + \dots
\end{aligned}
$$

- Invariance of order g: $S_0[\phi+\delta_1\phi] - S_0[\phi] + S_1[\phi, h+\delta_0 h] - S_1[\phi,h] = 0$
- Current [$\mathcal{J}$] generated cubic terms: $S_1[\phi,h] \sim \int \mathcal{J}[\phi]\ h$

$$S_0[\phi+\delta_1\phi] - S_0[\phi] \sim \int \mathcal{J}\ \delta_0 h \xrightarrow{\text{on-shell}} \text{conservation of } \mathcal{J}$$

Manifest Supersymmetry & Superspace description

- Apply Noether's method to supersymmetric theories
- matter fields → matter supermiltiplets (Φ, Σ)
  - Σ → complex linear superfield: $\bar{\mathrm{D}}^2\Sigma = 0$
  - Φ → chiral superfield: $\bar{\mathrm{D}}_{\dot\alpha}\Phi = 0$
- higher spin fields → higher spin supermultiplets

1. Integer superspin $Y = s$: $(j = s + 1/2, j = s)$

$$\Psi_{\alpha(s)\dot\alpha(s-1)} : \delta_0\Psi_{\alpha(s)\dot\alpha(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot\alpha(s-1)} + \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot\alpha_{s-1}}\Lambda_{\alpha(s)\dot\alpha(s-2))}$$

$$V_{\alpha(s-1)\dot\alpha(s-1)} : \delta_0 V_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s}\bar{L}_{\alpha(s-1)\dot\alpha(s)}$$

2. Half-integer superspin $Y = s + 1/2$: $(j = s + 1, j = s + 1/2)$

$$H_{\alpha(s)\dot\alpha(s)} : \delta_0 H_{\alpha(s)\dot\alpha(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot\alpha(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}L_{\alpha(s)\dot\alpha(s-1))}$$

$$\chi_{\alpha(s)\dot\alpha(s-1)} : \delta_0\chi_{\alpha(s)\dot\alpha(s-1)} = \bar{\mathrm{D}}^2 L_{\alpha(s)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot\alpha(s-1)}$$

Chiral constraint

Goal: Find a $\delta_1\Phi$ and use it for Noether's method

- $\delta_1\Phi$ is linear in (the derivatives of) Φ [ATTENTION: also in $\bar{\Phi}$]
- Chiral constraint: $\bar{\mathrm{D}}_{\dot{\alpha}}\delta_1\Phi = 0$
- Reminder: Linearized superdiffeomorphism

$$\delta_1\Phi = A^{\alpha}\,\mathrm{D}_{\alpha}\Phi + \Delta^{\alpha\dot{\alpha}}\,\partial_{\alpha\dot{\alpha}}\Phi$$

- Idea: Higher spin ansatz

$$\begin{aligned}\delta_1\Phi = \sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\Big\{ & A_l^{\alpha(k+1)\dot{\alpha}(k)}\,\Box^l\,\mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}_{\dot{\alpha}_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot{\alpha}_1}\mathrm{D}_{\alpha_1}\Phi \\ & +\Gamma_l^{\alpha(k)\dot{\alpha}(k+1)}\,\Box^l\,\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\mathrm{D}^2\bar{\mathrm{D}}_{\dot{\alpha}_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot{\alpha}_1}\mathrm{D}_{\alpha_1}\Phi \\ & +\Delta_l^{\alpha(k)\dot{\alpha}(k)}\,\Box^l\,\bar{\mathrm{D}}_{\dot{\alpha}_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot{\alpha}_1}\mathrm{D}_{\alpha_1}\Phi \\ & + E_l^{\alpha(k)\dot{\alpha}(k)}\,\Box^l\,\mathrm{D}^2\bar{\mathrm{D}}_{\dot{\alpha}_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot{\alpha}_1}\mathrm{D}_{\alpha_1}\Phi\Big\}\end{aligned}$$

"Higher Spin" transformation of Chiral superfield

- **Chirality**: $A^{l}_{\alpha(k+1)\dot\alpha(k)} = -\frac{k+1}{k+2}\,\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\Delta^{l}_{\alpha(k+1)\dot\alpha(k+1)}\,,$

$$\Gamma^{l}_{\alpha(k)\dot\alpha(k+1)} = \frac{1}{(k+1)!}\,\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\Delta^{l+1}_{\alpha(k)\dot\alpha(k))}\,,$$

$$E^{l}_{\alpha(k)\dot\alpha(k)} = \bar{\mathrm{D}}^{2}\Delta^{l+1}_{\alpha(k)\dot\alpha(k)}\,,$$

$$\bar{\mathrm{D}}_{(\dot\beta}\Delta^{0}_{\alpha(k)\dot\alpha(k))} = 0 \;\rightarrow\; \underline{\Delta^{0}_{\alpha(k)\dot\alpha(k)} = \tfrac{1}{k!}\bar{\mathrm{D}}_{(\dot\alpha_k}\ell_{\alpha(k)\dot\alpha(k-1))}}\,,$$

$$\bar{\mathrm{D}}_{\dot\beta}\Delta^{0} = 0 \;\rightarrow\; \underline{\Delta^{0} = \bar{\mathrm{D}}^{2}\ell}$$

- **Non-triviality**: $\Delta^{l+1}_{\alpha(k)\dot\alpha(k))},\; l = 0, 1, \ldots$ can be ignored

(trivial redefinitions of Φ)

- **Result**:

$$\delta_1\Phi = -\sum_{k=0}^{\infty}\Big\{\bar{\mathrm{D}}^{2}\ell^{\alpha(k+1)\dot\alpha(k)}\,\mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Phi$$

$$-\frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\,\bar{\mathrm{D}}_{\dot\alpha_{k+1}}\mathrm{D}_{\alpha_{k+1}}\ldots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Phi\Big\}$$

$$+\bar{\mathrm{D}}^{2}\ell\,\Phi$$

Higher Spin - Matter cubic interactions

- Matter theory: Single, free, massless chiral $S_0 = \int d^8z \, \bar{\Phi}\Phi$

$$\delta_g S_o = -g \int \sum_{k=0}^{\infty} \Big\{ \Big[\bar{\mathrm{D}}^2 \ell^{\alpha(k+1)\dot{\alpha}(k)} - \mathrm{D}_{\alpha_{k+2}} \lambda^{\alpha(k+2)\dot{\alpha}(k)} \Big] \mathrm{D}_{\alpha_{k+1}} \mathcal{T}_{\alpha(k)\dot{\alpha}(k)} + c.c.$$

$$+ \Big[\tfrac{1}{(k+1)!} \mathrm{D}^{(\alpha_{k+1}} \bar{\ell}^{\alpha(k))\dot{\alpha}(k+1)} - \tfrac{1}{(k+1)!} \bar{\mathrm{D}}^{(\dot{\alpha}_{k+1}} \ell^{\alpha(k+1)\dot{\alpha}(k))} \Big] \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)} \Big\}$$

$$-g \int \Big[\bar{\mathrm{D}}^2 \ell + \mathrm{D}^2 \bar{\ell} \Big] \mathcal{J}$$

$$\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)} = X^{(k+1)}_{\alpha(k+1)\dot{\alpha}(k+1)} + \frac{1}{(k+1)!} \Big[\mathrm{D}_{(\alpha_{k+1}} \bar{\mathrm{D}}^2 \bar{U}_{\alpha(k))\dot{\alpha}(k+1)} + c.c. \Big]$$

$$\mathcal{T}_{\alpha(k)\dot{\alpha}(k)} = i^k \partial^{(k)} \Phi \, \bar{\Phi} - \tfrac{k+2}{k+1} Z^{(k+1)}_{\alpha(k)\dot{\alpha}(k)} + \tfrac{k+2}{k+1} \mathrm{D}^{\alpha_{k+1}} U_{\alpha(k+1)\dot{\alpha}(k)} + \bar{\mathrm{D}}^{\dot{\alpha}_{k+1}} \bar{U}_{\alpha(k)\dot{\alpha}(k+1)}$$

$$\mathcal{J} = -\Phi \, \bar{\Phi} \qquad \Big[\, Z^{(k+1)}_{\alpha(k))\dot{\alpha}(k))} = (-i)^k \sum_{p=0}^{k} (-1)^p \, \partial^{(p)} \Phi \, \partial^{(k-p)} \bar{\Phi} \, \Big]$$

- Cubic interaction terms:

$$S = g \int \sum_{s=0}^{\infty} \Big\{ H^{\alpha(s)\dot{\alpha}(s)} \, \mathcal{J}_{\alpha(s)\dot{\alpha}(s)} + \Big[\chi^{\alpha(s)\dot{\alpha}(s-1)} \, \mathrm{D}_{\alpha_s} \mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} + c.c. \Big] \Big\}$$

Higher Spin Supercurrent multiplet

- Canonical supercurrent multiplet: $\left\{\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)}\ ,\ \mathcal{T}_{\alpha(k)\dot{\alpha}(k)}\right\}$
  - $\mathcal{T}_{\alpha(k)\dot{\alpha}(k)}$ → h.s. supertrace
  - $\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)}$ → h.s. supercurrent
- Metsaev bound : ✓ (cubic vertex includes higher derivatives)
- Only half-integer superspin supermultiplets $(s+1, s+1/2)$
- Conservation equations:

$$\bar{\mathrm{D}}^{\dot{\alpha}_{k+1}}\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)} = \frac{1}{(k+1)!}\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_{k+1}}\mathcal{T}_{\alpha(k))\dot{\alpha}(k)}\ ,\ k=0,1,2,\dots\ ,$$

$$\bar{\mathrm{D}}^2\mathcal{J} = 0$$

Minimal supercurrent multiplet

- There is one improvement term (U):

$$\mathcal{T}_{\alpha(k)\dot{\alpha}(k)} = i^k \partial^{(k)}\Phi\,\bar{\Phi} - \frac{k+2}{k+1} Z^{(k+1)}_{\alpha(k)\dot{\alpha}(k)} + \frac{k+2}{k+1} \mathrm{D}^{\alpha_{k+1}} U_{\alpha(k+1)\dot{\alpha}(k)} + \bar{\mathrm{D}}^{\dot{\alpha}_{k+1}} \bar{U}_{\alpha(k)\dot{\alpha}(k+1)}$$

- Minimal supercurrent multiplet : $\left\{ \mathcal{J}^{min}_{\alpha(k+1)\dot{\alpha}(k+1)} \ , \ 0 \right\}$ $\quad \underline{\mathcal{T}^{min}_{\alpha(k)\dot{\alpha}(k)} = 0}$

- Can minimal supercurrent multiplet be reached ? YES [property of S_0]

$$S_{\text{HS-}\Phi\text{ cubic interactions}} = g \sum_{s=0} \int H^{\alpha(s)\dot{\alpha}s}\ \mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)}$$

$$s=0\ :\ \mathcal{J}^{min} = -\Phi\,\bar{\Phi}$$

$$s=1\ :\ \mathcal{J}^{min}_{\alpha\dot{\alpha}} = \tfrac{1}{3}\,\mathrm{D}_\alpha\Phi\,\bar{\mathrm{D}}_{\dot{\alpha}}\bar{\Phi} + \tfrac{i}{3}\,\partial_{\alpha\dot{\alpha}}\Phi\,\bar{\Phi} - \tfrac{i}{3}\,\Phi\,\partial_{\alpha\dot{\alpha}}\bar{\Phi}$$

$$s=2\ :\ \mathcal{J}^{min}_{\alpha\beta\dot{\alpha}\dot{\beta}} = -\tfrac{1}{10}\,\partial^{(2)}\Phi\,\bar{\Phi} - \tfrac{1}{10}\,\Phi\,\partial^{(2)}\bar{\Phi} + \tfrac{2}{5}\,\partial\Phi\,\partial\bar{\Phi} - \tfrac{i}{5}\,\mathrm{D}\Phi\,\partial\bar{\mathrm{D}}\bar{\Phi} + \tfrac{i}{5}\,\partial\mathrm{D}\Phi\,\bar{\mathrm{D}}\bar{\Phi}$$

$$s=3\ :\ \mathcal{J}^{min}_{\alpha\beta\gamma\dot{\alpha}\dot{\beta}\dot{\gamma}} = -\tfrac{i}{35}\,\partial^{(3)}\Phi\,\bar{\Phi} + \tfrac{i}{35}\,\Phi\,\partial^{(3)}\bar{\Phi} + i\tfrac{9}{35}\,\partial^{(2)}\Phi\,\partial\bar{\Phi} - i\tfrac{9}{35}\,\partial\Phi\,\partial^{(2)}\bar{\Phi}$$
$$-\tfrac{3}{35}\,\partial^{(2)}\mathrm{D}\Phi\,\bar{\mathrm{D}}\bar{\Phi} - \tfrac{3}{35}\,\mathrm{D}\Phi\,\partial^{(2)}\bar{\mathrm{D}}\bar{\Phi} + \tfrac{9}{35}\,\partial\mathrm{D}\Phi\,\partial\bar{\mathrm{D}}\bar{\Phi}$$

- $\mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)} \sim$ Conformal higher spin supercurrent

$$\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = 0$$ [Kuzenko-Manvelyan-Theisen 2017]

Component higher spin currents

- Project superspace conservation equation to components
- Find only 3 independent components

$$\mathcal{J}^{\textit{min}\,(0,0)}_{\alpha(s)\dot{\alpha}(s)} \sim \mathcal{J}^{\textit{min}}_{\alpha(s))\dot{\alpha}(s))}\Big|_{\substack{\theta=0\\ \bar{\theta}=0}}$$

$$\mathcal{J}^{\textit{min}\,(1,1)(S,S)}_{\alpha(s+1)\dot{\alpha}(s+1)} \sim [\mathrm{D}_{(\alpha_{s+1}},\bar{\mathrm{D}}_{(\dot{\alpha}_{s+1}}]\mathcal{J}^{\textit{min}}_{\alpha(s))\dot{\alpha}(s))}\Big|_{\substack{\theta=0\\ \bar{\theta}=0}}$$

$$\mathcal{J}^{\textit{min}\,(1,0)(S)}_{\alpha(s+1)\dot{\alpha}(s)} \sim \mathrm{D}_{(\alpha_{s+1}}\mathcal{J}^{\textit{min}}_{\alpha(s))\dot{\alpha}(s))}\Big|_{\substack{\theta=0\\ \bar{\theta}=0}}$$

- Two bosonic currents and one fermionic current
- Satisfy usual (spacetime) conservation

Single, free, massive, chiral matter theory

- Consider a mass term correction:

$$S_0 = \int d^8z \; \bar{\Phi}\Phi \; + \frac{m}{2}\int d^6z \; \Phi^2 \; + \frac{m}{2}\int d^6\bar{z} \; \bar{\Phi}^2$$

- Add mass corrections to the minimal supercurrent multiplet:

$$\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = \mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)} + m\sum_{p=0}^{s-1}\gamma_p \, \partial^{(p)}\mathrm{D}\bar{\mathrm{D}}\Lambda \, \partial^{(s-1-p)}\Phi + m\sum_{p=0}^{s-1}\delta_p \, \partial^{(p)}\bar{\mathrm{D}}\Lambda \, \partial^{(s-1-p)}\mathrm{D}\Phi$$
$$- m\sum_{p=0}^{s-1}\gamma^*_p \, \partial^{(p)}\bar{\mathrm{D}}\mathrm{D}\bar{\Lambda} \, \partial^{(s-1-p)}\bar{\Phi} - m\sum_{p=0}^{s-1}\delta^*_p \, \partial^{(p)}\mathrm{D}\bar{\Lambda} \, \partial^{(s-1-p)}\bar{\mathrm{D}}\bar{\Phi}$$

$$\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} = \; 0 + m\sum_{p=0}^{s-1}\zeta_p \, \partial^{(p)}\Lambda \, \partial^{(s-1-p)}\Phi + m\sum_{p=0}^{s-1}\xi_p \, \partial^{(p)}\bar{\Lambda} \, \partial^{(s-1-p)}\bar{\Phi}$$
$$+ m\sum_{p=0}^{s-2}\sigma_p \, \partial^{(p)}\bar{\mathrm{D}}\mathrm{D}\Lambda \, \partial^{(s-2-p)}\Phi$$

- Check conservation equation: YES BUT ONLY FOR ODD s $(s = 2p+1)$

consistent coupling only with $(2p+2, 2p+3/2)$ supermultiplets

Nonlinear sigma model

- Supersymmetric nonlinear sigma model as a starting action:

$$S_0 = \int d^8z\, \mathcal{K}(\Phi,\bar{\Phi}) + \int d^6z\, \mathcal{W}(\Phi) + \int d^6\bar{z}\, \bar{\mathcal{W}}(\bar{\Phi})$$

$$\mathcal{K}(\Phi,\bar{\Phi}) \sim \mathcal{K}(\Phi,\bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}) \,,$$
$$\mathcal{W}(\Phi) \sim \mathcal{W}(\Phi) + \textit{constant}$$

- Is there a supercurrent multiplet to couple it to $(s+1, s+1/2)$ supermultiplet?

$$S_1 \sim \int d^8z \left\{ H^{\alpha(s)\dot{\alpha}(s)} \mathcal{J}_{\alpha(s)\dot{\alpha}(s)} + [\chi^{\alpha(s)\dot{\alpha}(s-1)} \mathrm{D}_{\alpha_s} \mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} + c.c.] \right\}$$

$$\bar{\mathrm{D}}^{\dot{\alpha}_s} \mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = \tfrac{1}{s!} \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_s} \mathcal{T}_{\alpha(s-1))\dot{\alpha}(s-1)}$$

Nonlinear sigma model: Results

- $s = 0$ (vector supermultiplet): ONLY IF $\mathcal{K} = \mathcal{K}(\Phi\bar{\Phi})$, $\mathcal{W}(\Phi) = 0$

$$\mathcal{J} = \Phi\mathcal{K}_\Phi \ , \ \bar{\mathrm{D}}^2\mathcal{J} = 0$$

[global U(1) $\xrightarrow{\text{gauging}}$ coupling to vector supermultiplet]

- $s = 1$ (supergravity supermultiplet): YES for any $\mathcal{K}$, $\mathcal{W}$

$$\mathcal{J}_{\alpha\dot{\alpha}} = \mathrm{D}_\alpha\Phi\,\bar{\mathrm{D}}_{\dot{\alpha}}\mathcal{K}_\Phi - \mathrm{D}_\alpha\bar{\mathrm{D}}_{\dot{\alpha}}(\Lambda\mathcal{F}) + \bar{\mathrm{D}}_{\dot{\alpha}}\mathrm{D}_\alpha(\bar{\Lambda}\bar{\mathcal{F}})$$

$$\mathcal{T} = -\mathcal{K} + \Lambda\mathcal{F} + 2\bar{\Lambda}\bar{\mathcal{F}} \qquad [\Phi = \bar{\mathrm{D}}^2\Lambda, \ \mathcal{W} = \Phi\mathcal{F}]$$

[Expected! Anything can be coupled to supergravity]

- $s \geq 2$ (higher spin supermultiplets): ONLY IF
  1.) $\mathcal{K} = \bar{\Phi}\Phi$, $\mathcal{W} = 0$: Free, massless, chiral superfield
  2.) $\mathcal{K} = \bar{\Phi}\Phi$, $\mathcal{W} = f\Phi$: Free, chiral superfield with linear superpotential
  3.) $\mathcal{K} = \bar{\Phi}\Phi$, $\mathcal{W} = m\Phi^2$: Free, massive, chiral superfield
- Results are consistent with: 1. Coleman-Mandula

  2. Maldacena-Zhiboedov

Variant matter supermultiplet: The complex linear

- Consider a complex linear supermultiplet for the matter theory:

$$S_0 = -\int d^8z\, \bar{\Sigma}\Sigma$$

- Repeat: Find $\delta_1 \Sigma \ \rightarrow \ \dots$
- OR use chiral - complex lineare duality:

$$S = -\int d^8z\, \bar{\sigma}\,\sigma + \int d^8z\, \Phi\,\sigma + \int d^8z\, \bar{\Phi}\,\bar{\sigma} + g_\sigma \int d^8z \sum_{s=0}^{\infty} H^{\alpha(s)\dot{\alpha}(s)} \mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)}$$

- $\mathcal{J}^{min}_{\text{complex linear }\alpha(s)\dot{\alpha}(s)} = \mathcal{J}^{min}_{chiral\ \alpha(s)\dot{\alpha}(s)}\Big|_{\Phi\rightarrow\Sigma}$
- $g_\Sigma = (-1)^{s+1} g_\Phi$
  1. For even spin $j = s+1$ (supergravity, ...) same charge
  2. for odd spin $j = s+1$ (vector supermultiplet, ...) opposite charge

Beyond matter theories

- Consider cubic vertices among higher spin supermultiplets
- Focus on non-minimal coupling (higher derivative lagrangians)
- Special type of vertices constructed by (super) field strengths
- Distinctive feature: Uniqueness (up to trivial field redefinitions)
  [Berends, Burgers, van Dame 1986] [Gelfond, Skvortsov, Vasiliev 2008]
- Superfield strengths of higher spin supermultiplets:

$$Y = s + 1/2: \quad \mathcal{W}_{\alpha(2s+1)} \sim \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{2s+1}} \partial_{\alpha_{2s}}{}^{\dot{\alpha}_s} \partial_{\alpha_{2s-1}}{}^{\dot{\alpha}_{s-1}} \dots \partial_{\alpha_{s+1}}{}^{\dot{\alpha}_1} H_{\alpha(s))\dot{\alpha}(s)}$$

$$Y = s: \quad \mathcal{W}_{\alpha(2s)} \sim \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{2s}} \partial_{\alpha_{2s-1}}{}^{\dot{\alpha}_{s-1}} \partial_{\alpha_{2s-2}}{}^{\dot{\alpha}_{s-2}} \dots \partial_{\alpha_{s+1}}{}^{\dot{\alpha}_1} \Psi_{\alpha(s))\dot{\alpha}(s-1)}$$

$$\bar{\mathrm{D}}_{\dot{\beta}} \mathcal{W}_{\alpha(k)} = 0 \ \text{(chiral)} \quad , \quad \mathrm{D}^{\alpha_k} \mathcal{W}_{\alpha(k)} = 0 \ \text{(e.o.m)} \ , \ k = 2s, 2s+1$$

$Y_1 - Y_2 - Y_2$

- Consider the cubic vertex $Y_1 - Y_2 - Y_2$ where $Y_1 = s_1 + 1/2$ and Y_2 is arbitrary
- Generated by a supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$

$$S_I = g \int d^8 z H^{\alpha(s_1)\dot{\alpha}(s_1)} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} \quad , \quad \bar{\mathrm{D}}^{\dot{\alpha}_{s_1}} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = 0$$

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = \bar{\mathcal{J}}_{\alpha(s_1)\dot{\alpha}(s_1)}$$

- General ansatz:

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = \sum_{p=0}^{s_1-2Y_2} \alpha_p \, \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)} \, \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} + \sum_{p=0}^{s_1-2Y_2-1} \beta_p \, \partial^{(p)} \mathrm{D}\mathcal{W}_{\alpha(2Y_2)} \, \partial^{(s_1-2Y_2-1-p)} \bar{\mathrm{D}}\bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$$

- Consequences of conservation equation:

$$\beta_p = -i\,(-1)^{2Y_2}\,\alpha_{p+1}\,\frac{p+1}{s_1-p} \;,\; p = 0, 1, \ldots, s_1 - 2Y_2 - 1$$

$Y_1 - Y_2 - Y_2$

- Consequences of reality:

$$\alpha_p = \alpha^*_{s_1-2Y_2-p} \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 \ ,$$

$$\beta_p = \beta^*_{s_1-2Y_2-1-p} \ , \ p = 0, 1, \ldots, s_1 - 2Y_2 - 1$$

- Unique supercurrent:

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = c(i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p}\binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \Big\{ \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)} \, \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$$

$$+ i(-1)^{2Y_2} \frac{s_1 - 2Y_2 - p}{2Y_2 + 1 + p} \partial^{(p)} \mathrm{D}\mathcal{W}_{\alpha(2Y_2)} \, \partial^{(s_1-2Y_2-1-p)} \bar{\mathrm{D}} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} \Big\}$$

- Constrained superspin values: $s_1 \geq 2Y_2$
  Supersymmetric extention of Weinberg-Witten theorem
- Special case (I): $s_1 = 2Y_2$, $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = c \, \mathcal{W}_{\alpha(2Y_2)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$
  Supersymmetric and higher spin extention of the Bel-Robinson tensor
  [Howe, Stelle, Townsend 1981]
- Special case (II): $Y_2 = 0$ (matter theory)
  Recover matter higher spin supercurrents

THANK YOU!