Spin-vortex radiation from relativistic charged particles

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Problem setting in the semi-classical relativistic radiation theory

Hamiltonian of the Dirac equation with regard to the anomalous magnetic moment of the electron:

$$\hat{H} = m_0 c - \imath \gamma_\mu \hat{P}^\mu - \frac{\mu_a}{2c} \sigma_{\alpha\beta} H^{\alpha\beta} \tag{1}$$

Here $\gamma^\mu = \imath\rho_3(1, \boldsymbol{\alpha})$ and $\sigma^{\alpha\beta} = (-\imath\boldsymbol{\alpha}, \boldsymbol{\sigma})$ are the Dirac matrices,

$$\sigma_{\alpha\beta} H^{\alpha\beta} = 2\left\{(\boldsymbol{\sigma}\mathbf{H}) - \imath(\boldsymbol{\alpha}\mathbf{E})\right\},$$

$\mu_a = \mu_0(g-2)/2$ is the anomalous magnetic moment of the electron,
$\mu_0 = e\hbar/2m_0 c$ is the Bohr magneton.

The operator of potential energy choosing the Coulomb gauge:

$$ {}^{e+\mu}\hat{U} = -e\left(\boldsymbol{\alpha}\tilde{\mathbf{A}}\right) - \mu_a\left\{\rho_3\left(\boldsymbol{\sigma}\tilde{\mathbf{H}}\right) + \rho_2\left(\boldsymbol{\sigma}\tilde{\mathbf{E}}\right)\right\} \tag{2} $$

The radiation fields satisfy the relations

$$ \tilde{\mathbf{A}} = \imath\frac{c}{\tilde{\omega}}\tilde{\mathbf{E}}, \qquad \tilde{\mathbf{H}} = -\imath\frac{\tilde{\omega}}{c}\left(\mathbf{n}\tilde{\mathbf{A}}\right) $$

Next, we will consider radiation using more clearly arranged methods of the semi-classical theory of radiation, according to which the trajectory of a particle can be described purely kinematically, and the radiation itself produces by quantum transitions with matrix elements on the basis of the wave functions of free Dirac particles.

Then

$$
\begin{aligned}
< {}^{e+\mu}\hat{U} >= -e\left(\beta\tilde{\mathbf{A}}\right)\left(1+\frac{\varepsilon}{2}\right) < \zeta'|\zeta > \\
-\mu_0 < \zeta'|\boldsymbol{\sigma'}\left\{\left(\frac{1}{\gamma}+a\right)\left[\beta\tilde{\mathbf{E}}\right]-\frac{\gamma}{\gamma+1}\beta\left(\beta\tilde{\mathbf{H}}\right)\right\}|\zeta >
\end{aligned}
\qquad (3)
$$

The first term corresponds to the charge radiation without a spin-flip. The constant $\varepsilon = \hbar\tilde{\omega}/E'$ takes into account the recoil effects with the finite energy $E' = m_0 c^2 \gamma'$ after radiation. The second one is according to the radiation of the intrinsic magnetic moment of a particle. For $\zeta' = \zeta$, it is a potential energy of intrinsic magnetic moment:

$$
< {}^{\mu}U >= -\mu_0\left(< \zeta|\boldsymbol{\sigma'}|\zeta > \mathbf{H_0}\right). \qquad (4)
$$

Also the spin precession in the rest frame can be obtained in terms of the magnetic field

$$\gamma \mathbf{H_0} = \gamma \left\{ \left(\frac{1}{\gamma} + a \right) \mathbf{H} - \frac{a\gamma}{\gamma+1} \beta \left(\beta \mathbf{H} \right) - \left(\frac{1}{\gamma+1} + a \right) [\beta \mathbf{E}] \right\},$$

since in this case the precession equation

$$\frac{d\zeta}{d\tau} = [\zeta \mathbf{H_0}]$$

coincides with the BMT equation in terms of proper time

$$\frac{d\zeta}{d\tau} = \frac{eg}{2m_0c}[\zeta \mathbf{H}] - \frac{g-2}{2} \frac{e}{m_0c} \frac{\gamma^2}{\gamma+1} [\zeta[\beta[\beta \mathbf{H}]]] - \frac{e\gamma}{m_0c} \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) [\zeta[\beta \mathbf{E}]]$$
$$= \frac{eg}{2m_0c}[\zeta \mathbf{H_0}] + \frac{e}{m_0c} \frac{\gamma}{\gamma+1} [\beta[\beta \mathbf{E_0}]] = \frac{e}{m_0c}[\zeta \mathbf{H_0}].$$

Thus, in the classical theory of relativistic radiation, the semi-classical potential energy can be obtained with the use of the spin BMT equation for the unit spin vector ζ in the rest frame

$$^{e+\mu}U = -e\left(\beta\widetilde{\mathbf{A}}\right) - \mu_0\left(\boldsymbol{\zeta}\left\{\left(\frac{1}{\gamma}+a\right)\mathbf{H} - \left(\frac{1}{\gamma+1}+a\right) - \frac{a\gamma}{\gamma+1}\beta(\beta\mathbf{H})\right\}\right). \quad (5)$$

Comparing expressions (3) and (5), one can derive the effective velocity

$$\begin{aligned}\beta^{\mathbf{eff}} = \left(1+\frac{\varepsilon}{2}\right)\beta < \zeta'|\zeta > -\imath\frac{\tilde{\omega}}{ec}\mu_0\left\{\left(\frac{1}{\gamma}+a\right)[< \boldsymbol{\sigma}' > \mathbf{n}]\right. \\ \left.\left(\frac{1}{\gamma+1}+a\right)[< \boldsymbol{\sigma}' > \beta] + \frac{a\gamma}{\gamma+1}(< \boldsymbol{\sigma}' > \beta)[\beta\mathbf{n}]\right\}.\end{aligned} \quad (6)$$

$$\beta^{\textit{eff}} = \beta_{\textit{e}} + \beta^{\textit{L}} + \beta^{\textit{Th}}, \tag{7}$$

$$\beta_{\textit{e}} = \left(1 + \frac{\varepsilon}{2}\right)\beta < \zeta'|\zeta >$$

corresponds to the radiation of a charge,

$$\beta^{\textit{L}} = -\imath\frac{\tilde{\omega}}{ec}\frac{g}{2}\mu_0\left\{)[< \boldsymbol{\sigma}' > (\mathbf{n} - \beta] - \frac{\gamma}{\gamma+1}(< \boldsymbol{\sigma}' > \beta)[\beta\mathbf{n}]\right\}$$

is according to the Larmor precession,

$$\beta^{\textit{Th}} = -\imath\frac{\tilde{\omega}}{ec}\frac{\gamma}{\gamma+1}\mu_0\left\{[< \boldsymbol{\sigma}' > \beta](1 - (\mathbf{n}\beta) + \beta[< \boldsymbol{\sigma}' > [\beta\mathbf{n}]\right\}$$

coheres with the Thomas precession.

Specificity of mixed charge and intrinsic magnetic moment radiation in the relativistic semi-classical radiation theory of the electron

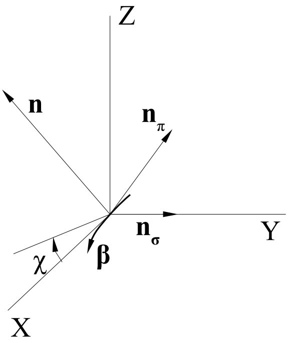

Figure: The coordinate system

$$\boldsymbol{\beta} = \left[1 - \frac{1}{2}\left(\frac{1}{\gamma^2} + \chi^2 + \frac{c^2t^2}{\rho^2}\right)\right]\mathbf{n} + \frac{ct}{\rho}\mathbf{n} - \chi\mathbf{n},$$

here

$$\mathbf{n} = \frac{\tilde{\mathbf{r}}}{\tilde{r}} = \left(1 - \frac{\chi^2}{2}, 0, \chi\right)$$

is the unit vector along the direction of the radiation toward the observer,

$$\mathbf{n}_\sigma = (0, 1, 0), \quad \mathbf{n}_\pi = \left(-\chi, 0, 1 - \frac{\chi^2}{2}\right)$$

are unit linear polarization vectors.

The Fourier transform of the electric field of electron radiation in neglect of the recoil effect:

$$ {}^{e+\mu}\tilde{\mathbf{E}}_{\tilde{\omega}} = \frac{\imath e\tilde{\omega}}{\tilde{r}c}\int_{-\infty}^{+\infty}\beta^{eff}\exp\imath\tilde{\omega}[t-(\mathbf{nr})/c]dt $$

$$ {}^{e}_{s}\tilde{E}_{\tilde{\omega}} = \left({}^{e}\tilde{E}_{\tilde{\omega}}\mathbf{n_s}\right) = \frac{\imath e\tilde{\omega}}{\tilde{r}c}\int_{-\infty}^{+\infty}(\boldsymbol{\beta}\mathbf{n_s})\left(1+\frac{\varepsilon}{2}\right) < \zeta'|\zeta > \exp\imath\tilde{\omega}[t-(\mathbf{nr})/c]dt $$

$$ {}^{\mu}_{s}\tilde{E}^{L}_{\tilde{\omega}} = \left({}^{\mu}\tilde{E}^{L}_{\tilde{\omega}}\mathbf{n_s}\right) = \frac{g}{2}\mu_0\frac{\tilde{\omega}^2}{\tilde{r}c^2}\int_{-\infty}^{+\infty}\left(<\boldsymbol{\sigma}'>\mathbf{N^L_s}\right)\exp\imath\tilde{\omega}[t-(\mathbf{nr})/c]dt, $$

$$ {}^{\mu}_{s}\tilde{E}^{Th}_{\tilde{\omega}} = \left({}^{\mu}\tilde{E}^{Th}_{\tilde{\omega}}\mathbf{n_s}\right) = \mu_0\frac{\tilde{\omega}^2}{\tilde{r}c^2}\int_{-\infty}^{+\infty}\left(<\boldsymbol{\sigma}'>\mathbf{N^{Th}_s}\right)\exp\imath\tilde{\omega}[t-(\mathbf{nr})/c]dt, $$

where

$$\mathbf{N}_{\mathbf{s}}^{\mathbf{L}} = [(\mathbf{n}-\boldsymbol{\beta})\mathbf{n}_{\mathbf{s}}] - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}\left([\boldsymbol{\beta}\mathbf{n}_{\mathbf{s}}]\mathbf{n}_{\mathbf{s}}\right),$$

$$\mathbf{N}_{\mathbf{s}}^{\mathbf{Th}} = (1-(\mathbf{n}\boldsymbol{\beta})\,[\boldsymbol{\beta}\mathbf{n}_{\mathbf{s}}] + (\boldsymbol{\beta}\mathbf{n}_{\mathbf{s}})[\boldsymbol{\beta}\mathbf{n}]$$

$$\left(<\boldsymbol{\sigma}'>\mathbf{N}_{\sigma}^{\prime\mathbf{L}}\right) = \frac{1-\zeta\zeta'}{2}\frac{g}{2}\frac{1}{2}\left(\frac{1}{\gamma^2}-\chi^2+\frac{c^2t^2}{\rho^2}\right)\zeta,$$

$$\left(<\boldsymbol{\sigma}'>\mathbf{N}_{\pi}^{\prime\mathbf{L}}\right) = \frac{1+\zeta\zeta'}{2}\frac{g}{2}\left(-\frac{ct}{\rho}\chi\right)\zeta,$$

$$\left(<\boldsymbol{\sigma}'>\mathbf{N}_{\sigma}^{\prime\mathbf{Th}}\right) = \frac{1+\zeta\zeta'}{2}\frac{g}{2}\frac{1}{2}\left(\frac{1}{\gamma^2}+\chi^2-\frac{c^2t^2}{\rho^2}\right)\zeta,$$

$$\left(<\boldsymbol{\sigma}'>\mathbf{N}_{\pi}^{\prime\mathbf{Th}}\right) = \frac{1+\zeta\zeta'}{2}\frac{ct}{\rho}\chi\zeta.$$

For

$$\mathbf{N_s} = \mathbf{N_s^L} + \mathbf{N}_s^{Th},$$

we have

$$(<\boldsymbol{\sigma}' > \mathbf{N}_\sigma) = \frac{1+\zeta\zeta'}{2}\left[\left(\frac{g}{2}+1\right)\frac{1}{\gamma^2}\right] - \left(\frac{g}{2}-1\right)\left(\chi^2 - \frac{c^2t^2}{\rho^2}\right)\frac{1}{2}\chi,$$

$$(<\boldsymbol{\sigma}' > \mathbf{N}_\pi) = \frac{1+\zeta\zeta'}{2}\left(\frac{g}{2}-1\right)\left(-\frac{ct}{\rho}\right)\chi\zeta.$$

It is interesting that when $g = 2$, the Larmor and Thomas precession of the spin after addition give the same contribution to the σ-component, and the π-component of the radiation is completely absent.

Integral characteristics of mixed radiation with allowance for spin-vortex radiation

Spectral-angular distribution of the power of polarized radiation:

$$\frac{d^{e+\mu}W_s}{d\tilde{\omega}d\Omega} = \frac{\tilde{r}^2 c^2}{8\pi^3\rho}|{}^e\tilde{\mathbf{E}}_\mathbf{s} + {}^\mu\tilde{\mathbf{E}}_\mathbf{s}|^2$$

$$\frac{d^{e\mu}W_\sigma}{d\tilde{\omega}d\Omega} = \frac{e^2}{6\pi^3}\frac{\hbar\tilde{\omega}^3\rho\zeta}{m_0c^4\gamma^5}(1+\psi^2)^{3/2}\left[\frac{g}{2}\psi^2 - (1+\psi^2)\right]K_{1/3}(x)K_{2/3}(x),$$

$$\frac{d^{e\mu}W_\pi}{d\tilde{\omega}d\Omega} = \frac{e^2}{6\pi^3}\frac{\hbar\tilde{\omega}^3\rho\zeta}{m_0c^4\gamma^5}(1+\psi^2)^{3/2}\left(\frac{g}{2} - 1\right)\psi^2 K_{1/3}(x)K_{2/3}(x).$$

The terms that contain g-factor is according to Larmor precession, and another ones correspond to Thomas precession.

Taking into account these expressions,

$$\tilde{\omega}^3 d\tilde{\omega} = \frac{3^4 c^4}{\rho^4} \frac{\gamma^{12}}{(1+\psi^2)^6} \chi^3 d\chi,$$

$$\int_0^{+\infty} x^3 K_{1/3}(x) K_{2/3}(x) = \frac{35\pi^2}{2^5 3^3},$$

we get angular distribution of the power of mixed radiation

$$\frac{d^{e\mu} W_\sigma}{d\psi} = \frac{35}{2^5} \frac{e^2 \hbar \zeta}{m_0 \rho^3} \gamma^6 \frac{[(\frac{g}{2} - 1)\psi^2 - 1]}{(1+\psi^2)^{3/2}},$$

$$\frac{d^{e\mu} W_\pi}{d\psi} = \frac{35}{2^5} \frac{e^2 \hbar \zeta}{m_0 \rho^3} \gamma^6 \frac{(\frac{g}{2} - 1)\psi^2}{(1+\psi^2)^{3/2}},$$

The total power of polarized $e\mu$-radiation:

$$ {}^{e\mu}W_\sigma = W_{SR}\left(\frac{g}{2}-7\right)\frac{1}{6}\xi\zeta, \quad {}^{e\mu}W_\pi = W_{SR}\left(\frac{g}{2}-1\right)\frac{1}{6}\xi\zeta, $$

where

$$ W_{SR} = \frac{2}{3}\frac{e^2c}{\rho^2}\gamma^4, \quad \xi = \frac{3}{2}\frac{\hbar\gamma^2}{m_0 c\rho}. $$

$$ {}^{e\mu}W = W_{SR}\left(\frac{g}{2}-4\right)\frac{1}{3}\xi\zeta, $$

and at $g=2$ we get

$$ {}^{e\mu}W|_{g=2} = -W_{SR}\xi\zeta $$

Thank you for your attention!