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Two dimensional Wess–Zumino models as gauge theories

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A "toy" model : The Schwinger model : massless QED_2

A not so toy model : $\mathcal{N}=2$ supersymmetric QED_2

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Supersymmetry : hidden in plain sight

- Supersymmetry was invented for very specific reasons. But its scope has grown. Its realization, within the framework of the Standard Model, is but one example of its relevance for physics and doesn't exhaust its relevance. The purpose of this talk is to show how it can be useful for providing insights to a much broader class of physical models, by providing effective strategies for describing non-perturbative effects through lattice techniques.

Supersymmetry : hidden in plain sight

- In particular, we wish to stress that supersymmetry provides the only consistent definition of quantum field theory, beyond perturbation theory, since it expresses the fact that "dynamical system+fluctuations" is consistently closed and doesn't depend on how the separation between the two is realized.
  This closure is realized by describing the dynamics through the Langevin equation.
  How can this be applied to gauge theories ? Let's start with two–dimensional gauge theories.

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Why two–dimensional gauge theories ?

- They're of obvious interest as string backgrounds
- They've become of "practical" interest in materials science.
- They're accessible to numerical techniques, that allow going way beyond the constraints of analytical results.
- They don't propagate gauge fields ! So they're scalar models in disguise ! So it's possible to use the results from studying scalar models and apply them to gauge theories. In particular, define a Langevin equation for describing the fluctuations.
- Where do they differ from scalar models ? In the interpretation–and the names–of the different quantities, in particular the auxiliary fields.

The subtleties of the Schwinger model

The Schwinger model, massless QED_2, seems like an interacting theory :

$$\mathscr{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \mathrm{i}\overline{\psi} \not{\mathscr{D}} \psi$$

In fact everything about it can be described by the theory of a *massive* scalar field

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 - \frac{e^2}{2}\phi^2$$

However this, seemingly trivial, theory, can't be described by a Langevin equation ! It doesn't describe a closed system, but one in interaction with a bath, whose degrees of freedom can't be resolved.

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No Langevin formulation for the one–flavor Schwinger model

The reason is that there's *one* scalar field–but two worldvolume dimensions ! So it's not possible to write the Langevin equation that does lead to a theory with target space Lorentz invariance (and target space supersymmetry)

$$\eta_I = \sigma^A_{IJ}\frac{\partial\phi_J}{\partial u_A} + \frac{\partial W}{\partial\phi_I}$$

with

$$\{\sigma_A, \sigma_B\} = 2\delta_{AB}$$

(in Euclidian signature)–a field's missing ! So a way to describe a "consistently closed" system is by the *two*–flavor Schwinger model, whose closure would be $\mathcal{N} = 2$ QED_2. $\mathcal{N} = 2$ QED_2.

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The illusion

One way to describe this theory is by its Lagrangian, in components :

$$\mathscr{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \mathrm{i}\overline{\lambda}\partial\!\!\!/\lambda - \frac{D^2}{2} + \mathscr{L}_{\mathrm{matter}}(\psi, \overline{\psi}, \phi, F)$$

to which we may add two, additional, terms :

- The $\theta-$term :

$$\mathscr{L}_\theta = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}F_{\mu\nu}$$

- The Fayet–Iliopoulos term

$$\mathscr{L}_{\mathrm{FI}} = \xi D$$

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The above Lagrangian *seems* to describe a vector multiplet, coupled to a chiral multiplet, with propagating degrees of freedom. In 1+1 dimensions this is an illusion, since the gauge field doesn't propagate! Charged matter *might* propagate, under certain conditions.
The gauge field properties are "hidden" in the auxiliary fields.

Where the gauge field is hidden

The content of the gauge theory can, thus, be repackaged in a gauge invariant superfield (called the "twisted chiral superfield")

$$\Sigma = \sigma + \theta\lambda + \overline{\theta\lambda} + \theta\overline{\theta}(D - \mathrm{i}E)$$

The electric field and the auxiliary field of the vector multiplet become the imaginary/real part of the auxiliary field ; and the dynamics is controlled by the superpotential $W(\Sigma)$ and the "twisted superpotential"

$$\widetilde{W}(\Sigma) = \mathrm{i}\tau\Sigma$$

with

$$\tau \equiv r + \mathrm{i}\frac{\theta}{2\pi}$$

The twisted superptential, in fact, defines the linear terms–that determine the boundary conditions !

Repackaging as the $\mathcal{N} = 2$ Wess–Zumino model

So, if it is possible to focus on the dynamics of the σ, one has an $\mathcal{N} = 2$ WZ model, where the $\theta-$term and the FI term define the boundary conditions.
The equations for the *noise* fields

$$\eta_1 = \frac{\partial W}{\partial \sigma_1} + \xi \quad \text{and} \quad \eta_2 = \frac{\partial W}{\partial \sigma_2} - E$$

lead to several interesting cases :

- $\langle \sigma \rangle \neq 0$, $\langle \eta \rangle = 0$: The "Higgs branch"–global target space Lorentz invariance is broken, supersymmetry, however, isn't.
- $\langle \sigma \rangle = 0$, $\langle \eta \rangle \neq 0$: The "Coulomb branch"–global target space Lorentz invariance apparently intact, supersymmetry broken.

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- The FI term and the $\theta-$term describe the *linear* terms in the superpotential, that define the boundary conditions and are related to D–brane configurations. It is their backreaction that can restore target space Lorentz invariance, while breaking supersymmetry.
- We remark that the $\theta-$term and the FI term are components of the same auxiliary field–hence an ambiguity in deciding which is which.

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- It is possible to describe consistently the fluctuations, to which physical systems are coupled, by quantum, thermal effects, or disorder, in a way that doesn't depend on how the physical degrees of freedom are selected among the fluctuations–and the reason is that the fluctuations and the physical degrees of freedom are but components of the same supermultiplet. The fluctuations are mutually non–local wrt the physical degrees of freedom.

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- The difference between quantum, thermal and disorder fluctuations is that, while, in the latter two cases, the effective anticommuting degrees of freedom can be expressed in terms of more fundamental degrees of freedom, that can be commuting at that scale, quantum fluctuations cannot be expressed as local, commuting, degrees of freedom–but as local, anticommuting, degrees of freedom, only.

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The most direct building blocks of a quantum field theory, therefore, seem to be the noise fields

$$\eta^I(\tau^A) = s_A^{IJ}\frac{\partial\phi^J}{\partial\tau^A} + \frac{\partial W}{\partial\phi_I} \equiv s_A^{IJ}\frac{\partial\phi^J}{\partial\tau^A} + F^I$$

From the known properties of the noise it is possible to deduce the properties of the physical fields and, in particular, describe how the bath reacts to the presence of the physical degrees of freedom. A consistent reaction is described by the fact that the 1–point function can take a non–zero value ; and that the potential can change, due to tunneling effects.

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Two–dimensional gauge theories don't propagate gauge fields–they propagate scalars and spinors and it is supersymmetry that ensures that the system is consistently "closed" : that the correlation functions don't depend on how the dynamical degrees of freedom are picked out from the fluctuations. The wealth of exact, mathematical, results serve to calibrate the numerical methods, that can go much more easily where analytical tools aren't very effective–a case in point is in recovering the effects of the fermionic determinant–along with its phase–implicitly, through the identities satisfied by the correlation functions of the noise fields and the scalars, using the action of the scalars as *input*. It is, by no means, the action, of the full quantum theory, that is the *output* of the calculations and captures the effects of the instantons, for instance, in full generality.

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A very nice consistency check that the fermionic determinant is correctly taken into account–including its sign and/or phase–is by computing the 2–point correlation function

$$\langle(\eta_I(u) - \langle\eta_I(u)\rangle)\,(\eta_J(u') - \langle\eta_J(u')\rangle)\rangle \equiv$$
$$\left\langle\left(\sigma_A^{IK}\frac{\partial\phi_K}{\partial u_A} + \frac{\partial W}{\partial\phi_I}\right)\left(\sigma_B^{JL}\frac{\partial\phi_L}{\partial u_B} + \frac{\partial W}{\partial\phi_J}\right)\right\rangle \propto \delta_{IJ}\delta(u-u')$$

and checking that it is, indeed, ultra–local, on the lattice. These are, therefore, concrete quantities and identities, to which numerical techniques can be applied.

Conclusions–Perspectives

Interesting question : What's the best way to probe the metric of the target space of the scalar components of the (twisted) chiral superfield be reconstructed from the fluctuations described by the Langevin equation, when the chiral matter superfield can't be integrated out ? Might a generalization of the Langevin equation along the lines of

$$E_I^A(\phi)\eta_A = \sigma_a^{IJ}\frac{\partial \phi_J}{\partial u_A} + \frac{\partial W}{\partial \phi_I}$$

be useful ? This is the typical equation for describing the fluctuations of the magnetization of nanomagnets, and is used to describe real experiments–where it's known as the Landau–Lifshitz–Gilbert equation.

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The non–trivial question might be how the fluctuations of the vielbeine themselves are to be described. The answer is that supersymmetry ensures that the metric takes into account the backreaction to its probes.
In fact this is too simplistic and a way to sample Kähler metrics in an effectively has been proposed by S. Donaldson (2005) and many checks have been carried out by F. Ferrari and by A. Bilal and their students more recently and work is in progress to develop it into a more general computational tool.

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The bottom line : Supersymmetry isn't optional : it's an *inevitable* property of the consistent description of any consistently closed system and expresses the independence of how dynamical degrees of freedom are labeled from the fluctuations.
The real question isn't how it's broken–it's how it's realized. Computational techniques have made it possible to eliminate many analytical limitations and experiments in condensed matter systems have become sensitive to the susceptibilities, whose relations can probe the superpartners.

Conclusions–Perspectives

Many happy returns,
Iosif Lvovich, Peter Mikhailovich and Kelly !
May we always celebrate together new insights !