Unimodular (and dilaton) quantum gravity

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Outline

Five formulations of GR: EG

$$S_{\mathrm{EG}}(g) = Z_N \int d^4x \, \sqrt{|g|} \, R \qquad \text{where} \qquad Z_N = \frac{1}{16\pi G} \, .$$

Invariant under *Diff M*

Five formulations of GR: DG

Apply Stückelberg trick to EG.

$$
\begin{aligned}
S_{\mathrm{DG}}(g,\chi) &= S_{\mathrm{EG}}\left(\frac{1}{Z_N}\chi^2 g_{\mu\nu}\right) \\
&= \int d^4x\,\sqrt{|g|}\left[\chi^2 R - 6\chi\nabla^2\chi\right]
\end{aligned}
$$

Invariant under *Diff* $\ltimes$ *Weyl*

$$
g_{\mu\nu} \to \Omega^2 g_{\mu\nu}\ , \qquad \chi \to \Omega^{-1}\chi\ .
$$

Five formulations of GR: UG

$$\sqrt{|g|} = \omega$$

$$S_{\mathrm{UG}}(g) = Z_N \int d^4x \, \omega \, R \, .$$

Invariant under *SDiff M*

Five formulations of GR: UD

$$S_{\mathrm{UD}}(g,\chi) = \int d^4x\,\omega\,\left[\chi^2 R - 6\chi\nabla^2\chi\right]\,.$$

Invariant under $Diff^*M$

Five formulations of GR: *WTDiff*

$$\begin{aligned} S_X(g_{\mu\nu}) &= S_{UG}\left(g_{\mu\nu}, \chi_0\left(\frac{|g|}{\omega^2}\right)^{-\frac{1}{8}}\right) \\ &= S_{EG}\left(\left(\frac{|g|}{\omega^2}\right)^{-1/4} g_{\mu\nu}\right) \\ &= Z_N \int d^4x\, |g|^{\frac{1}{4}}\, \omega^{\frac{1}{2}} \left[R + \frac{3}{32}\left(|g|^{-1}\nabla|g| - 2\omega^{-1}\nabla\omega\right)^2\right] \end{aligned}$$

Invariant under *SDiff M* $\ltimes$ *Weyl*

Five formulations of GR: summary

$$\begin{tikzcd}
& \begin{array}{c}\textbf{DG}\\ (g,\chi);\ \mathit{Diff}\ltimes\mathit{Weyl}\end{array} \arrow[dl, "\chi=\chi_0"'] \arrow[d, "\chi=\chi_0\left(\frac{|g|}{\omega^2}\right)^{-\frac{1}{8}}"'] \arrow[dr, "\sqrt{|g|}=\omega"] & \\
\begin{array}{c}\textbf{EG}\\ (g);\ \mathit{Diff}\end{array} \arrow[dr, "\sqrt{|g|}=\omega"'] & \begin{array}{c}\mathit{WTDiff}\\ (g);\ \mathit{SDiff}\ltimes\mathit{Weyl}\end{array} \arrow[d, "\sqrt{|g|}=\omega"'] & \begin{array}{c}\textbf{UD}\\ (g_U,\chi);\ \mathit{Diff}^*\end{array} \arrow[dl, "\chi=\chi_0"] \\
& \begin{array}{c}\textbf{UG}\\ (g_U);\ \mathit{SDiff}\end{array} &
\end{tikzcd}$$

Hamiltonian formulation

	DG	**EG**	**UG**
fields	q_{ij},N_i,N,χ	q_{ij},N_i,N	q_{ij},N_i
momenta	p^{ij},P^i,P,π	p^{ij},P^i,P	p^{ij},P^i
# can. variables	22	20	18
primary constr.	P^i, P, C	P^i, P	P^i
secondary constr.	$\mathcal{H}_i$,$\mathcal{H}$	$\mathcal{H}_i$,$\mathcal{H}$	$\mathcal{H}_i$,$\mathcal{H}_\Lambda$
# 1st cl. constr.	9	8	7
# canonical d.o.f.	4	4	4

R. De Leon Ardon, S. Gielen, R. P., arXiv:1805.11626 [gr-qc]

Which is best?

Gauge invariances needed in order to deal with local d.o.f. UG has the smallest gauge group compatible with locality.

On the other hand:

1. extending the gauge group is useful to recognize equivalences between different formulations
2. larger gauge group means that certain singular configurations could only be gauge artifacts (e.g. big bang)
3. suggest route to unification

Quantum question

Can we maintain these equivalences in the quantum theory?

Check at one loop

York variables

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \left(\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{d} \bar{g}_{\mu\nu} \bar{\nabla}^2 \right) \sigma + \frac{1}{d} \bar{g}_{\mu\nu} h ,$$

$$\bar{\nabla}^\mu h^{\mathrm{TT}}_{\mu\nu} = 0 , \quad \bar{g}^{\mu\nu} h^{\mathrm{TT}}_{\mu\nu} = 0 , \quad \bar{\nabla}^\mu \xi_\mu = 0 , \quad h = \bar{g}^{\mu\nu} h_{\mu\nu} .$$

$$J_1 = \det \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right)^{1/2} \det \left(\Delta_{L0} \right)^{1/2} \det \left(\Delta_{L0} - \frac{\bar{R}}{d-1} \right)^{1/2}$$

$$\begin{aligned}
\Delta_{L0} \chi &= -\bar{\nabla}^2 \chi , \\
\Delta_{L1} A_\mu &= -\bar{\nabla}^2 A_\mu + \bar{R}_\mu{}^\rho A_\rho , \\
\Delta_{L2} h_{\mu\nu} &= -\bar{\nabla}^2 h_{\mu\nu} + \bar{R}_\mu{}^\rho h_{\rho\nu} + \bar{R}_\nu{}^\rho h_{\mu\rho} - \bar{R}_{\mu\rho\nu\sigma} h^{\rho\sigma} - \bar{R}_{\mu\rho\nu\sigma} h^{\sigma\rho} .
\end{aligned}$$

Transformations under *Diff*

$$\delta h_{\mu\nu} = \bar{\nabla}_\mu \epsilon_\nu + \bar{\nabla}_\nu \epsilon_\mu$$

$$\epsilon^\mu = \epsilon^{T\mu} + \bar{\nabla}_\mu \phi \; ; \qquad \bar{\nabla}_\mu \epsilon^{T\mu} = 0 \; .$$

$$\delta_{\epsilon^T} \xi^\mu = \epsilon^{T\mu} \; ; \qquad \delta_\phi h = -2\Delta_{L0}\phi \; ; \qquad \delta_\phi \sigma = 2\phi \; ,$$

$h^{TT}_{\mu\nu}$ and $s = h + \Delta_{L0}\sigma$ are invariant.

ξ_μ and $\psi = \frac{((d-1)\Delta_{L0} - \bar{R})\sigma + \beta h}{(d-1-\beta)\Delta_{L0} - \bar{R}}$ are gauge d.o.f.

One-loop EG

$$S = \frac{Z_N}{2}\int d^dx\,\sqrt{\bar{g}}\left\{\frac{1}{2}h^{\mathrm{TT}}_{\mu\nu}\left(\Delta_{L2}-\frac{2\bar{R}}{d}\right)h^{\mathrm{TT}\mu\nu} -\frac{(d-1)(d-2)}{2d^2}s\left(\Delta_{L0}-\frac{\bar{R}}{d-1}\right)s-\frac{d-2}{4d}Eh^2\right\}.$$

where EOM implies

$$0 = E \equiv \bar{R} - \frac{2d\Lambda}{d-2}$$

Gauge fixing

$$\begin{aligned}
F_\mu &= \bar\nabla_\rho h^\rho{}_\mu - \frac{\beta+1}{d}\bar\nabla_\mu h \\
&= -\left(\Delta_{L1} - \frac{2\bar R}{d}\right)\xi_\mu - \frac{d-1-\beta}{d}\nabla_\mu\left(\Delta_{L0} - \frac{\bar R}{d-1-\beta}\right)\psi
\end{aligned}$$

$$\begin{aligned}
S_{GF} &= \; = \frac{Z_N}{2\alpha}\int d^dx\sqrt{\bar g}\,\bar g^{\mu\nu}F_\mu F_\nu \\
&= \frac{Z_N}{2\alpha}\int d^dx\sqrt{\bar g}\Bigg[\xi_\mu\left(\Delta_{L1} - \frac{2\bar R}{d}\right)^2\xi^\mu \\
&\qquad + \frac{(d-1-\beta)^2}{d^2}\psi\Delta_{L0}\left(\Delta_{L0} - \frac{\bar R}{d-1-\beta}\right)^2\psi\Bigg]
\end{aligned}$$

Ghosts

$$C_\nu = C^T_\nu + \nabla_\nu \frac{1}{\sqrt{-\bar{\nabla}^2}} C^L$$

$$\begin{aligned} S_{gh} \;&=\; \int d^d x \sqrt{\bar{g}} \Big[\bar{C}^{T\mu} \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right) C^T_\mu \\ &\qquad + 2\frac{d-1-\beta}{d} \bar{C}^L \left(\Delta_{L0} - \frac{\bar{R}}{d-1-\beta} \right) C^L \Big] \end{aligned}$$

1-loop EG with cosmological term

$$Z_{GR}(\bar{g}) = e^{-S(\bar{g})} \int (d\epsilon) \, \frac{\mathrm{Det}_1 \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right)^{1/2}}{\mathrm{Det}_2 \left(\Delta_{L2} - \frac{2\bar{R}}{d} \right)^{1/2}}$$

$$V_{Diff} = \int (d\epsilon)$$

on shell $\bar{R} = \frac{2d\Lambda}{d-2}$

1-loop EG with cosmological term

$$\Gamma(\bar{g}) = S(\bar{g}) + \frac{1}{2}\mathrm{Tr}\log\left(\Delta_{L2} - \frac{2\bar{R}}{d}\right) - \frac{1}{2}\mathrm{Tr}\log\left(\Delta_{L1} - \frac{2\bar{R}}{d}\right)$$

$$\Gamma_{log}(\bar{g}) \; = \; -\frac{1}{2(4\pi)^2}\int d^4x\,\sqrt{\bar{g}}\log\left(\frac{\Lambda^2}{\mu^2}\right)\left(\frac{53}{45}\bar{R}_{\mu\nu\rho\sigma}\bar{R}^{\mu\nu\rho\sigma} - \frac{29}{40}\bar{R}^2\right)$$

S.M. Christensen, M.J. Duff Nucl. Phys. B170 (1980) 480-506

Quantum UG

Usual linear splitting $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ does not fit well with unimodularity condition.
Use instead exponential splitting:

$$g_{\mu\nu} = \bar{g}_{\mu\rho} \left(e^X \right)^{\rho}{}_{\nu}$$

where $\det \bar{g} = \omega$ and $X^{\rho}{}_{\nu} = \bar{g}^{\rho\sigma} h_{\sigma\nu}$.
Symmetric in spite of appearance.
Can use York decomposition on $h_{\mu\nu}$ as before.
On shell results same for EG.

Gauge fixing for *SDiff*

SDiff is generated by transverse vector fields

$$\bar{\nabla}_\mu \epsilon^\mu = 0$$

$$F_\mu = T_{\mu\nu} \bar{\nabla}_\rho h^{\rho\nu} = -\left(\Delta_{L1} - \frac{2\bar{R}}{d} \right) \xi_\mu$$

where

$$T^\mu{}_\nu = \delta^\mu_\nu - \bar{\nabla}^\mu \frac{1}{\bar{\nabla}^2} \bar{\nabla}_\nu$$

$$S_{GF} = \frac{Z_N}{2\alpha} \int d^d x\, \omega F_\mu T^{\mu\nu} F_\nu = \frac{Z_N}{2\alpha} \int d^d x\, \omega\, \xi_\mu \left(-\bar{\nabla}^2 - \frac{\bar{R}}{d} \right)^2 \xi^\mu$$

$$S_{gh} = \int d^d x\, \omega\, \bar{C}^T_\mu \left(-\bar{\nabla}^2 - \frac{\bar{R}}{d} \right) C^{\mu\,T}$$

The partition function of unimodular gravity is

$$Z_{UG} = e^{-S(\bar{g})} \left(\int (d\epsilon^T) \right) \frac{\mathrm{Det}_1 \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right)^{1/2}}{\mathrm{Det}_2 \left(\Delta_{L2} - \frac{2\bar{R}}{d} \right)^{1/2} \mathrm{Det}\Delta_{L_0}^{1/2}}$$

Volume of *SDiff*

$$(d\epsilon) = (d\epsilon^T)(d\phi)\mathrm{Det}\Delta_{L_0}^{1/2}$$

Since $\delta_\phi h = -2\Delta_{L0}\phi$, the measure on $Q = Diff/SDiff$ is

$$(dh) = (d\phi)\,\mathrm{Det}\Delta_{L0}\ .$$

Thus

$$V_{Diff} = \int (d\epsilon) = \int (d\epsilon^T)\,\det\Delta_{L_0}^{-1/2} \int (d\phi)\,\det\Delta_{L0} = V_{SDiff}\,V_Q\ ,$$

where

$$V_{SDiff} = \int (d\epsilon^T)\mathrm{Det}\Delta_{L_0}^{-1/2}$$

Alternatively

$$\begin{aligned} V_{SDiff} &= \int (d\epsilon)\delta(\bar{\nabla}^{\mu}\epsilon_{\mu}) \\ &= \int (d\epsilon^{T})(d\phi)\mathrm{Det}\Delta_{L0}^{1/2}\delta(\Delta_{L0}\phi) \\ &= \int (d\epsilon^{T})\mathrm{Det}\Delta_{L0}^{-1/2} \end{aligned}$$

In conclusion

$$Z_{UG} = e^{-S(\bar{g})} V_{SDiff} \frac{\mathrm{Det}_1 \left(\Delta_{L1} - \frac{2\bar{R}}{d} \right)^{1/2}}{\mathrm{Det}_2 \left(\Delta_{L2} - \frac{2\bar{R}}{d} \right)^{1/2}}$$

and

$$\Gamma^{(1)}_{UG}(g) = \Gamma^{(1)}_{EG}(g)\Big|_{\det g = \omega} + \mathrm{constant}$$

Quantum DG

Equivalence maintained if Weyl invariance maintained.

Regularization breaks Weyl invariance, but its effect can be offset if dilaton is present

Englert, C. Truffin and R. Gastmans, Nucl. Phys. B117, 407 (1976)
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M. Shaposhnikov and D. Zenhäusern, Phys. Lett. B671, 162 (2009)
M. Shaposhnikov and I. Tkachev, Phys. Lett. B675, 403 (2009)
R. P., New J. Phys. **13** 125013 (2011) arXiv:1110.6758 [hep-th]
A. Codello, G. D'Odorico, C. Pagani, R. P., Class. Quant. Grav. 30 (2013), arXiv:1210.3284 [hep-th]

Z. Lalak and P. Olszewski, arXiv:1807.09296

Example: matter in background metric

Here $g_{\mu\nu}$ is fixed background

$$S(\phi; g_{\mu\nu}) = \frac{1}{2}\int d^4x\sqrt{g}\,\phi\Delta^{(1/6)}\phi\ , \quad \Delta^{(1/6)} = -\Box + \frac{R}{6}$$

Similar discussion for massless fermions and Maxwell fields

Standard scalar measure and path integral

$$(d\phi)^{\mathrm{I}} = \prod_x \frac{d\phi(x)}{\mu}$$

$$\Gamma^{\mathrm{I}}[g_{\mu\nu}] = -\log \int (d\phi) e^{-\int d^4x \sqrt{g}\phi\Delta^{(1/6)}\phi} = \frac{1}{2}\mathrm{Tr}\log\left(\frac{\Delta^{(1/6)}}{\mu^2}\right)$$

Trace anomaly

$$\delta_\omega \Gamma^{\mathrm{I}}[g_{\mu\nu}] = \int dx \sqrt{g}\, \omega(x) \langle T^\mu{}_\mu \rangle$$

$$\langle T^\mu{}_\mu \rangle = \frac{2}{\sqrt{g}} g^{\mu\nu} \frac{\delta \Gamma^{\mathrm{I}}}{\delta g^{\mu\nu}} = b\, C^2 + b'\, E$$

$E = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$
$C^2 = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$

$$b = \frac{1}{120(4\pi)^2}$$

$$b' = -\frac{1}{360(4\pi)^2}$$

The Weyl-invariant scalar measure

$$(d\phi)^{\mathrm{II}} = \prod_x \frac{d\phi(x)}{\chi(x)}$$

$$\Gamma^{\mathrm{II}}[g_{\mu\nu},\chi] = -\log \int (d\phi) e^{-\int d^4x\sqrt{g}\phi\Delta^{(1/6)}\phi} = \frac{1}{2}\mathrm{Tr}\log\left(\frac{1}{\chi^2}\Delta^{(1/6)}\right)$$

$$\frac{1}{\Omega^{-2}\chi^2}\Delta^{(1/6)}_{\Omega^2 g}(\Omega^{-1}\phi) = \Omega^{-1}\left(\frac{1}{\chi^2}\Delta^{(1/6)}\phi\right)$$

eigenvalues are Weyl invariant → $\mathrm{Det}\left(\frac{1}{\chi^2}\Delta^{(1/6)}\right)$ is Weyl invariant

Weyl invariant quantization

μ has been promoted to a field

dilaton acts as compensator (Stückelberg) field in the *quantum* effective action

Note: trace of energy-momentum tensor still nonzero:

$$\begin{aligned} 0 = \delta_\omega \Gamma^{\mathrm{II}}[g_{\mu\nu}, \chi] &= \int dx \sqrt{g} \left(\frac{\delta \Gamma^{\mathrm{II}}}{\delta g_{\mu\nu}} \delta_\omega g_{\mu\nu} + \frac{\delta \Gamma^{\mathrm{II}}}{\delta \chi} \delta_\omega \chi \right) \\ &= \int dx \sqrt{g}\, \omega(x) \left(\langle T^\mu{}_\mu \rangle - \frac{\delta \Gamma^{\mathrm{II}}}{\delta \chi} \chi \right) \end{aligned}$$

Another point of view

$$\Gamma^{\mathrm{I}}(g^{\chi}_{\mu\nu}) - \Gamma^{\mathrm{I}}(g_{\mu\nu}) = \Gamma_{WZ}(g_{\mu\nu}, \chi)$$

Wess-Zumino consistency condition:

$$\Gamma_{WZ}(g^{\Omega}_{\mu\nu}, \chi^{\Omega}) - \Gamma_{WZ}(g_{\mu\nu}, \chi) = -\Gamma_{WZ}(g_{\mu\nu}, \Omega)$$

where $g^{\Omega} = \Omega^2 g$, $\chi^{\Omega} = \Omega^{-1}\chi$
If we identify $\Gamma^{\mathrm{I}}(g_{\mu\nu})$ with $\Gamma^{\mathrm{II}}(g_{\mu\nu}, \chi = \mu)$,

$$\Gamma^{\mathrm{II}}(g_{\mu\nu}, \chi) = \Gamma^{\mathrm{I}}(g_{\mu\nu}) + \Gamma_{WZ}(g_{\mu\nu}, \chi)$$

Return to DG

Let $\delta g_{\mu\nu} = h_{\mu\nu}$, $\delta\chi = \eta$.

$$\begin{aligned} S^{(2)} &= \frac{1}{2}\mathcal{H}((h,\eta),(h,\eta)) \\ &= \frac{1}{2}\int d^4x\sqrt{g}\begin{pmatrix} h_{\mu\nu} & \eta \end{pmatrix}\begin{pmatrix} \mathcal{H}_{hh}^{\mu\nu\rho\sigma} & \mathcal{H}_{h\eta}^{\mu\nu} \\ \mathcal{H}_{\eta h}^{\rho\sigma} & \mathcal{H}_{\eta\eta} \end{pmatrix}\begin{pmatrix} h_{\rho\sigma} \\ \eta \end{pmatrix} \end{aligned}$$

$$
\begin{aligned}
\mathcal{H}_{hh}^{\mu\nu\rho\sigma} =& \frac{1}{12}\chi^2 \Bigg[-\frac{1}{2}\mathbf{1}^{\mu\nu\rho\sigma} D^2 + g^{(\nu|\sigma} D^{|\mu)} D^\rho - \frac{1}{2} g^{\mu\nu} D^\rho D^\sigma - \frac{1}{2} g^{\rho\sigma} D^{(\mu} D^{\nu)} \\
& + \frac{1}{2} g^{\mu\nu} g^{\rho\sigma} D^2 - \mathcal{R}^{\mu\rho\nu\sigma} - g^{(\mu|\rho}\mathcal{R}^{|\nu)\sigma} + \frac{1}{2}\left(g^{\mu\nu}\mathcal{R}^{\rho\sigma} + \mathcal{R}^{\mu\nu} g^{\rho\sigma}\right) \\
& + \left(\mathcal{R} - 12\lambda Z \chi^2\right) K^{\mu\nu\rho\sigma} \Bigg] \\
\mathcal{H}_{h\eta}^{\mu\nu} =& \mathcal{H}_{\eta h}^{\mu\nu} = \frac{1}{6}\chi \left(g^{\mu\nu} D^2 - D^\mu D^\nu + \mathcal{R}^{\mu\nu} - \frac{1}{2}\mathcal{R} g^{\mu\nu} \right) + 2\lambda Z \chi^3 g^{\mu\nu} \\
\mathcal{H}_{\eta\eta} =& D^2 - \frac{1}{6}\mathcal{R} + 12\lambda Z \chi^2 \ .
\end{aligned}
$$

with D_μ a Weyl-covariant derivative.

Wave operators

$$
\begin{aligned}
(\mathcal{O}_{hh})_{\mu\nu}{}^{\rho\sigma} &= \chi^{-4} g_{\mu\alpha} g_{\nu\beta} \mathcal{H}_{hh}^{\alpha\beta\rho\sigma} \ , \\
(\mathcal{O}_{h\eta})_{\mu\nu} &= \chi^{-4} g_{\mu\alpha} g_{\nu\beta} \mathcal{H}_{h\eta}^{\alpha\beta} \ , \\
\mathcal{O}_{\eta h}^{\rho\sigma} &= \chi^{-2} \mathcal{H}_{\eta h}^{\rho\sigma} \ , \\
\mathcal{O}_{\eta\eta} &= \chi^{-2} \mathcal{H}_{\eta\eta} \ .
\end{aligned}
$$

Weyl covariance of wave operators

$$
\begin{aligned}
(\mathcal{O}_{hh(\Omega^2 g_{\mu\nu},\Omega^{-1}\chi)})_{\mu\nu}{}^{\rho\sigma}(\Omega^2 h_{\rho\sigma}) &= \Omega^2(\mathcal{O}_{hh(g_{\mu\nu},\chi)})_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma} \ , \\
(\mathcal{O}_{h\eta(\Omega^2 g_{\mu\nu},\Omega^{-1}\chi)})_{\mu\nu}(\Omega^{-1}\eta) &= \Omega^2(\mathcal{O}_{h\eta(g_{\mu\nu},\chi)})_{\mu\nu}\eta \ , \\
(\mathcal{O}_{\eta h(\Omega^2 g_{\mu\nu},\Omega^{-1}\chi)})^{\rho\sigma}(\Omega^2 h_{\rho\sigma}) &= \Omega^{-1}(\mathcal{O}_{\eta h(g_{\mu\nu},\chi)})^{\rho\sigma} h_{\rho\sigma} \ , \\
\mathcal{O}_{\eta\eta(\Omega^2 g_{\mu\nu},\Omega^{-1}\chi)}(\Omega^{-1}\eta) &= \Omega^{-1}\mathcal{O}_{\eta\eta(g_{\mu\nu},\chi)}\eta \ .
\end{aligned}
$$

Gauge fixing

$$S_{GF} = \frac{1}{2\alpha} \int d^4x \sqrt{g}\, \frac{1}{2} \xi \chi^2 F_\mu \bar{g}^{\mu\nu} F_\nu \ ,$$

where

$$F_\nu = D_\mu h^\mu{}_\nu - \frac{\beta + 1}{4} D_\nu h$$

$$S_{gh} = \int d^4x \sqrt{g}\, \bar{C}_\nu \mathcal{O}^\nu_{gh\mu} C^\mu$$

$$(\mathcal{O}_{gh})^\nu_\mu = -\frac{1}{\chi^2} \left(\delta^\nu_\mu D^2 + \frac{1-\beta}{2} D_\mu D^\nu + \mathcal{R}_\mu{}^\nu \right)$$

Weyl-gauge-fixing $\eta = 0$

Weyl invariance of one loop effective action

$$\Gamma^{(1)}_{DG}(g_{\mu\nu},\chi) = S_{DG}(g_{\mu\nu},\chi) + \frac{1}{2}\mathrm{Tr}\log\mathcal{O}_{hh} - \mathrm{Tr}\log\mathcal{O}_{gh}$$

If $\Delta_{(g_{\mu\nu},\chi)}h_{\mu\nu} = \lambda h_{\mu\nu}$ then

$$\Delta_{(\Omega^2 g_{\mu\nu},\Omega^{-1}\chi)}\left(\Omega^2 h_{\mu\nu}\right) = \Omega^2\Delta_{(g_{\mu\nu},\chi)}h_{\mu\nu} = \lambda\Omega^2 h_{\mu\nu}$$

The spectrum of Δ is Weyl invariant, so $\Gamma^{(1)}$ is Weyl invariant.

Furthermore in gauge χ =constant, $\mathcal{O}_{hh} = \Delta_{L2} - \frac{2\bar{R}}{d}$
and therefore $\Gamma^{(1)}_{DG} = \Gamma^{(1)}_{EG}$

Stückelberg gauging commutes with quantization

$$\begin{tikzcd}
S_{EG}(g_{\mu\nu}; Z_N) \arrow[r] \arrow[d] & \Gamma_{EG}(g_{\mu\nu}; Z_N; \mu) \arrow[d] \\
S_{DG}(g_{\mu\nu}, \chi) \arrow[r] & \Gamma_{DG}(g_{\mu\nu}, \chi; u)
\end{tikzcd}$$

$Z_N \to \chi^2$

$\mu \to u\chi$

Conclusions

- Equivalences can be maintained at one loop
- extend to all orders
- extend to different actions
- UG interesting because of different role of vacuum energy. This extends to quantum UG.