Gravitational Properties of the Proca Field

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We study a 2-parameter family of Proca-Einstein models:

$$\mathcal{L} = \sqrt{-g}\,\Big[\,\tfrac{1}{2}M_P^2 R - \tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}m^2 A^2 - \tfrac{1}{2}\alpha G_{\mu\nu}A^\mu A^\nu + \tfrac{1}{4}\beta\Lambda^{-2}L_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\,\Big]$$

- Quadratic in Proca field, linear in curvature
- Minimal (modulo ambiguity) & quadrupole couplings
- All Proca-graviton-Proca cubic couplings included
- Ghost free: second-derivative equations of motion
- From naturalness: $|\alpha| \lesssim \mathcal{O}(1), \quad |\beta| \sim \mathcal{O}(1).$

Outline

Motivations

Estimation of UV Cutoff

Constraints from Shock-wave Analysis

Constraints from Unitarity & Analyticity

Remarks

Motivations

- Gravitational interactions are unavoidable
- Massive low-spin fields in flat space: No issues
- Ghost-free interacting theories exist
- Attractive for cosmology model building/ astrophysics
- Embedding in sensible UV completion???
- Parameter space must be constrained

UV Cutoff Estimation

- Analysis in Stückelberg formalism: $A_\mu = B_\mu - m^{-1}\partial_\mu \phi.$
- System acquires "gauge" symmetry (Stückelberg invariance):

$$\delta B_\mu = \partial_\mu \lambda, \qquad \delta\phi = m\lambda$$

- Gauge fixing: propagators smooth in massless limit

$$\Delta\mathcal{L} = -\tfrac{1}{2}\sqrt{-g}\left(\nabla_\mu B^\mu - m\phi\right)^2$$

- Canonically normalized graviton: $g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{\rm P}} h_{\mu\nu}$
- UV cutoff easily read from higher-dimensional operators

Case I, $\alpha \neq 0$, without Counter Terms

- Lowest suppression scale (no quadrupole): $\Lambda_3 \equiv \sqrt[3]{m^2 M_{\rm P}}$
- Inclusion of quadrupole term means: $\Lambda \gtrsim \Lambda_3$
- Take the following scaling limit:

$$m \to 0, \quad M_P \to \infty, \quad \Lambda_3 = \text{constant}$$

- Lagrangian, modulo field redefinitions, reduces to:

$$\mathcal{L}_{\rm g.f.} \to \frac{1}{2} B_\mu \partial^2 B^\mu + \frac{1}{2}\phi \partial^2 \phi + \frac{1}{2} h_{\mu\nu} \mathcal{G}^{\mu\nu} - \frac{\alpha^2}{8\Lambda_3^6} \partial_\mu \phi \, \partial_\nu \phi \, \mathcal{E}^{\mu\nu\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi$$

- UV cutoff reads: $\Lambda \sim \dfrac{\Lambda_3}{\sqrt[3]{|\alpha|}}\,, \quad 0 < |\alpha| \lesssim \mathcal{O}(1)$

Case II, $\alpha = 0$, with Counter Terms

- Lowest suppression scale pushed higher, $\Lambda_2 \equiv \sqrt{m M_{\rm P}}$, by:

$$\mathcal{L}_{\rm c.t.} = \tfrac{1}{8}\kappa\alpha^2 M_P^{-2}\sqrt{-g}\, A_\mu A_\nu \hat{\mathcal{E}}^{\mu\nu\rho\sigma} A_\rho A_\sigma, \qquad \kappa = 1$$

- Take the following scaling limit:

$$m \to 0, \qquad M_P \to \infty, \qquad \Lambda_2 = \text{constant}$$

- Lagrangian, modulo field redefinitions, reduces to:

$$\mathcal{L}_{\rm g.f.} + \mathcal{L}_{\rm c.t.} \to \tfrac{1}{2} B_\mu \partial^2 B^\mu + \tfrac{1}{2}\phi\partial^2\phi + \tfrac{1}{2} h_{\mu\nu}\mathcal{G}^{\mu\nu} + \sum_{n=1}^{\infty}\left(\frac{\alpha}{\Lambda_2^4}\right)^n \left(\mathcal{O}_{4n+4} + \beta\Lambda^{-2}\mathcal{O}_{4n+6}\right)$$

- UV cutoff reads: $\Lambda \sim \dfrac{\Lambda_2}{\sqrt[4]{|\alpha|}}, \qquad 0 < |\alpha| \lesssim \mathcal{O}(1)$

Case III, $\alpha = 0$

- Lagrangian acquires ***U(1)*** gauge invariance in massless limit
- There will be no "***1/m***" dependency
- Only Planck-suppressed higher dimensional operators
- UV cutoff is: $\Lambda \sim M_P\,, \quad \alpha = 0$
- This is model independent upper bound on the UV cutoff

- To be more precise:

$$\Lambda \sim \begin{cases} \frac{\Lambda_3}{\sqrt[3]{|\alpha|}}, & \mathcal{O}((m/M_P)^2) \lesssim |\alpha| \lesssim \mathcal{O}(1), \text{ without counter terms,} \\ \frac{\Lambda_2}{\sqrt[4]{|\alpha|}}, & \mathcal{O}((m/M_P)^2) \lesssim |\alpha| \lesssim \mathcal{O}(1), \text{ with counter terms,} \\ M_P, & |\alpha| \lesssim \mathcal{O}((m/M_P)^2). \end{cases}$$

Shock-Wave Analysis

- The Proca-Einstein system admits pp-wave solution:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} = \eta_{\mu\nu} + \mathcal{F}(u, \vec{x})\, l_\mu l_\nu$$

$$A_\mu = \bar{A}_\mu = M_P \mathcal{H}(u, \vec{x})\, l_\mu$$

- Provided that the functions satisfy:

$$\partial^2 \tilde{\mathcal{F}} = 0\,, \quad \left(\partial^2 - m^2\right) \mathcal{H} = 0$$

with $\quad \tilde{\mathcal{F}} \equiv \mathcal{F} + \left(1 + \alpha + 2\beta\Lambda^{-2}m^2 - \frac{1}{2}\beta\Lambda^{-2}\partial^2\right) \mathcal{H}^2$

- The solutions at $\vec{x} \neq 0$

$$\tilde{\mathcal{F}} = -\mathcal{A}(u) \ln(\Lambda|\vec{x}|)\,, \quad \mathcal{H} = \mathcal{B}(u) K_0(m|\vec{x}|)$$

- Taking the singularity at $\vec{x}=0$ into account:

$$M_P^{-2}T_{\mu\nu} = \pi l_\mu l_\nu \mathcal{A}(u)\delta^2(\vec{x})$$
$$+m^2 l_\mu l_\nu \mathcal{B}^2(u)\left[K_0^2(m|\vec{x}|)+K_1^2(m|\vec{x}|)\right]\left[1+\alpha-\beta\,\mathfrak{F}(m|\vec{x}|)\right]$$

with $\mathfrak{F}(m|\vec{x}|) \equiv \frac{1}{2}\Lambda^{-2}\left[K_0^2(m|\vec{x}|)+K_1^2(m|\vec{x}|)\right]^{-1}\partial^2 K_1^2(m|\vec{x}|) - m^2\Lambda^{-2}$

- Imposing null energy condition:

$$\mathcal{A}(u) \geq 0, \quad 1+\alpha-\beta\,\mathfrak{F}(m|\vec{x}|) \geq 0$$

- Within the EFT validity regime, we conclude

$$1+\alpha-2a\beta \geq 0, \quad \text{for } a \in (0,1]$$

- Violation of NEC may lead to superluminal propagation

- Linear Proca fluctuations in probe limit: $w_\mu = A_\mu - \bar{A}_\mu$
- Divergence of EoM gives constraint (3 DoF's)
- Take ***v*** as light-cone time, $\boldsymbol{w_u}$ as non-dynamical
- Choose "sandwich wave" profile:

$$\begin{pmatrix} \mathcal{A}(u) \\ \mathcal{B}(u) \end{pmatrix} = \begin{pmatrix} \mathcal{A}_0 \\ \mathcal{B}_0 \end{pmatrix} [1 - \theta(u^2 - \lambda^2)] \exp\left[-\frac{\lambda^2 u^2}{(u^2 - \lambda^2)^2}\right]$$

- So, $\mathcal{A}(u), \mathcal{B}(u) \in \mathrm{C}^\infty(\mathbb{R})$, with a compact support $[-\lambda, \lambda]$
- Ansatz for fluctuations: $w_\mu(u, v, \vec{x}) = \tilde{w}_\mu(u)\, e^{i(pv + \vec{q}\cdot\vec{x})}$
- Thin sandwich wave: Ignore change in transverse position during the course of the sandwich wave

- Work in the following parametric regime:

$$\Lambda \gtrsim \frac{1}{\lambda} \gg p \gg q \gg \frac{1}{b} \gg m$$

- Change in ***q*** during the course of the wave can be ignored
- Transverse position and momentum chosen to be aligned
- Redefine dynamical modes as:

$$\Phi_1 = \tilde{w}_v \,, \qquad \Phi_2 = \delta_{ij} e_i \tilde{w}_j \,, \qquad \Phi_3 = \varepsilon_{ij} e_i \tilde{w}_j$$

i,j = 1,2 indices on transverse plane, $\boldsymbol{e_i}$ momentum unit vector

- Equations of motion take the schematic form:

$$(\partial_u - ip\gamma)\,\Phi_I(u) = ip\left(\mathcal{A}(u)\mathcal{C}_{IJ} + \mathcal{B}^2(u)\mathcal{D}_{IJ}\right)\Phi_J(u)$$

where $\gamma \equiv \frac{1}{4}(q^2 + m^2)/p^2$ and $\mathcal{C}, \mathcal{D}$ are 3×3 matrices

- Choose $\mathcal{A}_0 = 0,\ \mathcal{B}_0 = \pm 1$ and diagonalize the system
- Integrate across the sandwich wave:

$$\Psi_I(+\lambda) \approx \Psi_I(-\lambda) \exp\left[ip \int_{-\lambda}^{+\lambda} \mathrm{d}u \, \left[\gamma + d_I \mathcal{B}^2(u)\right]\right]$$

- Integral is the ***v***-shift suffered by the ***I***-th mode
- Time delay relative to massless propagation in flat space:

$$\Delta v_I \approx 1.13\, d_I \lambda$$

- The eigenvalues of matrix $\mathcal{D}$ are:

$$d_1 \approx \alpha \left(1 + \alpha - 2\beta(\Lambda b)^{-2}\right) (mb)^{-2}$$
$$d_2 \approx d_3 \approx (1+\alpha) \ln^2(mb)$$

- NEC plus absence of negative time delay:

$$\alpha \geq 0, \quad \beta \leq \tfrac{1}{2}(1+\alpha)$$

Unitarity & Analyticity Constraints

- Local, unitary, analytic, Lorentz invariant UV completion
- Scattering amplitudes must satisfy certain inequalities
- Simple positivity constraints for crossing-symmetric forward scattering amplitudes
- Assumptions:

(i) compliance of high-energy amplitudes with optical theorem

(ii) Froissart bound

(iii) standard analyticity properties of S-matrix

- Perturbative UV completion (tree level diagrams suffice)

- On-shell 4-point scattering amplitudes of the Proca field
- ***t***-channel crossing symmetry requires to consider … …
- Same polarization of particles within pairs (1,3) and (2,4)
- Introduce IR regulator $\boldsymbol{\mu}$ to address t-channel singularity
- Kinematic regime of interest:

$$\Lambda \gg \sqrt{s} \gg \sqrt{-t} = \mu \ll m$$

- IR regulated forward limit crossing-symmetric amplitudes

$$\mathcal{M}_{ij}(s) = \mathcal{M}_{\lambda_i\lambda_j;\lambda_i\lambda_j}\left(s, t \to -\mu^2\right), \quad i, j = 0, 1, 2,$$

where $\boldsymbol{\lambda_i}$'s are physical polarizations

- Consider the quantity: $f_{ij} \equiv \frac{1}{2\pi i}\oint_{\mathcal{C}} ds\, \frac{\mathcal{M}_{ij}(s)}{(s-s_0)^3}$
- s_0 is in $(0,\ 4m^2)$ real-line segment on complex ***s***-plane
- From analytic dispersion relations:

$$f_{ij} = -\operatorname*{Res}_{s=-\infty}\left[\frac{\mathcal{M}_{ij}(s)}{(s-s_0)^3}\right]_{\text{EFT}}$$

- From Froissart bound, optical theorem, crossing symmetry

$$f_{ij} > 0, \quad i,j = 0,1,2$$

- Deforming contour, dropping boundary contributions, encircling multi-particle branch cuts starting at $s_0 = 0$ and $s_0 = 4m^2$.

- The results are:

$$f_{00} = (mM_P)^{-2}\left[\,(m/\mu)^2 + \alpha\,\right],$$

$$f_{10} = \tfrac{3}{8}(mM_P)^{-2}\alpha^2(1-\kappa) = f_{20},$$

$$f_{11} = (\Lambda M_P)^{-2}\left[\,(\Lambda/\mu)^2 + \tfrac{1}{2}\beta^2\,(m/\Lambda)^2 - \beta\,\right],$$

$$f_{22} = (\Lambda M_P)^{-2}\left[\,(\Lambda/\mu)^2 + \tfrac{1}{2}\beta^2\,(m/\Lambda)^2 - 3\beta\,\right],$$

$$f_{12} = (\Lambda M_P)^{-2}\left[\,(\Lambda/\mu)^2 + \tfrac{1}{2}\beta^2\,(m/\Lambda)^2 - \beta(2+\alpha)\right] = f_{21}$$

- Most $\boldsymbol{f_{ij}}$ dominated by ***t***-channel graviton exchange
- From $\boldsymbol{f_{10} = f_{20} > 0}$, we obtain: $\alpha \neq 0, \quad \kappa < 1$
- Model-independent UV cutoff upper bound changed to:

$$\Lambda \sim \Lambda_3/\sqrt[3]{|\alpha|}$$

Summary

- UV cutoff of EFT parametrically smaller than Planck mass

- Shock-wave analysis: null-energy condition and absence of negative time delays in high energy scattering puts constraints

- Unitarity-Analyticity constrains
  (i) ambiguity term must be present!
  (ii) cutoff upper bound much below Planck scale

- Combining all results:

$$\alpha > 0, \quad \beta \leq \tfrac{1}{2}(1+\alpha)$$

Thank You
for Your Attention