Constrained higher spin fields within BRST approach: Lagrangian formulations, interactions, Feynman rules

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Based on research with J.Buchbinder
A.R. arxiv:1803.04678[hep-th]; arxiv:1803.05173[hep-th]; J.Buchbinder, A.R., in progress

Talk is dedicated to the 70th anniversaries of Professors P. Lavrov and J. Buchbinder

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Outline

- Motivations: (un)constrained HS formulations on $\mathbb{R}^{1,d-1}$, (A)dS$_d$, SFT; BV Lagrangian quantization rules
- What are BRST-BFV and BRST-BV approaches to derive of (un)constrained Lagrangians for HS fields?
- What problems to solve: unknown methods to derive LF and BV actions for HS fields?
- 2 equivalent ways to get Constrained BRST-BFV LF for (half-)integer HS fields on $R^{1,d-1}$: Reduction & self-consistency;
- Constrained GI Lagrangians for (half)-integer HS fields with $Y(s_1, ...s_k)$;
- From Constrained BRST-BFV Lagrangian formulation to Constrained BRST-BV for (half)-integer HS fields. Case integer spin $Y(s_1)$;
- BRST-BV approach for Lagrangian formulations: minimal BV action, antibracket, Δ_H on Fock-space $\mathcal{H}_{tot}$;
- On consistent interaction from minimal BRST-BV Lagrangian;
- Extension of minimal BRST-BFV (BRST-BV) to non-minimal BRST-BFV (BRST-BV) for HS fields;
- Fock-space based Quantum master equation, gauge fixing for constrained $Y(s_1)$ Bose fields;
- Generating functionals of Green functions, BRST transforms, Ward idents;

Motivations: (un)constrained HS formulations on $\mathbb{R}^{1,d-1}$, (A)dS$_d$, SFT

(M. Fierz, W. Pauli; V. Ginzburg; E.Fradkin; L. Singh, C. Hagen; C. Fronsdal, M.Vasiliev) the problems of HS field theory have attracted & attracting the significant attention as one from directions of the research in LHC to find new matter & interactions, (TS for $k=1$ row in Young tableau $Y(s_1,...,s_k)$), (MS $k>1$) $\mathbf{s}=(s_1,s_2,...)$ (massive and massless:$m=0$) HS fields :

$$\Phi_{(\mu)_{s_1},(\nu)_{s_2},...,(\rho)_{s_k}} \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|}\hline \mu_1 & \mu_2 & \cdot \quad \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{s_1} \\ \hline \nu_1 & \nu_2 & \cdot \quad \cdot \quad \cdot & \cdot & \cdot & \nu_{s_2} \\ \cline{1-6} \cdots & \cdots & \cdots \quad \cdot \quad \cdot & \cdot & \cdots \\ \cline{1-5} \end{array} = Y(s_1,...,s_k)\,.$$

in view of connection to SuperString Field Theory (SFT): (E. Witten (1986); C. Thorn(1989)) through special tensionless limit for intercept ($\alpha' \to \infty$): (A. Sagnotti, M. Tsulaia, (2004)).

$\Longrightarrow$SFT $\stackrel{\alpha'\to\infty}{\longrightarrow}$ $\{\infty\}$ set of HS fields in s/string spectrum

As shown, Tsulaia & Sagnotti (2004) the free dynamic for massless TS, MS HS fields follows from the tensionless limit for free open bosonic strings leads to triplet-like LF and after imposing of appropriate off-shell constraints on the field and gauge parameters to the Fronsdal formulation for integer HS fields (on $\mathbb{R}^{1,d-1}$) in terms of $\Phi^{(\mu)_s}$, $\delta\Phi^{(\mu)_s} = -\partial^{\{\mu_s}\xi^{(\mu\})_{s-1}}$ **for** $\Phi'' = 0$ **and** $\xi' = 0$ - *holonomic off-shell constraints.*
Tensionless limit: (for $[a_{\mu_k}, a^+_{\nu_l}] = -\delta_{kl}\eta_{\mu_k\nu_l}$, $\mu_k = 0, 1, ..., d-1$, $k, l \in \mathbb{N}_0$, $\mathrm{diag}\eta_{\mu\nu} = (+, -, ..., -)$)

$$\lim_{\alpha'\to\infty} Q_V \; = Q = \eta_0 l_0 + \sum_{k>0}^{\infty}\Big[\eta_k^+ l_k + \eta_k l_k^+ + \imath\eta_k^+\eta_k\mathcal{P}_0\Big] = \eta_0 l_0 - \imath\mathcal{P}_0 M + \Delta Q,$$

$$\text{for } \Big(l_k,\, l_k^+,\, l_0\Big) = \Big(-ia^{\mu_k}\partial_{\mu_k},\, -ia^{+\mu_k}\partial_{\mu_k},\, \partial^2\Big); \; \boxed{\{\eta_k^+, \mathcal{P}_l\} = \delta_{kl}, \{\eta_0,\, \mathcal{P}_0\} = \imath};$$

$$\overset{Virasoro}{algebra}: \; \lim_{\alpha'\to\infty}[L_k, L_l^+] = [l_k, l^+ l_l] = l_0\delta_{lk}$$

$Q^2 = 0 \; \forall \; d$ as compared to $Q_V^2 = 0$, when $d = 26$ (1983).

Motivations: SFT $\rightarrow$ reducible bosonic HS fields

The String Field equation, for vacuum $(\mathcal{P}_0, \mathcal{P}_k, \eta_k)|0\rangle = 0$, $k > 0$ in $D = 26$
$\rightarrow Q^2 = 0$ being subject to tower of reducible gauge symmetries,

$$Q_V|\Phi\rangle = 0,\ \delta|\Phi\rangle = Q_V|\Lambda_0\rangle,\ \delta|\Lambda_0\rangle = Q_V|\Lambda_1\rangle, \ldots \quad (1)$$

turn in tensionless limit after expanding as Q the string field $|\Phi\rangle$, $|\Lambda_0\rangle$, ...

$$|\Phi\rangle = |\varphi_1\rangle + \eta_0|\varphi_2\rangle, \quad |\Lambda_k\rangle \ = |\Lambda^0_{1k}\rangle + \eta_0|\Lambda^1_k\rangle, \ldots \quad (2)$$

in terms of η_0-independent equations

$$l_0|\varphi_1\rangle - \Delta Q|\varphi_2\rangle = 0, \quad \Delta Q|\varphi_1\rangle - M|\varphi_2\rangle = 0, \quad (3)$$

$$\delta|\varphi_1\rangle \ = \Delta Q|\Lambda^0_{10}\rangle - M|\Lambda^1_0\rangle, \quad \delta|\varphi_2\rangle \ = l_0|\Lambda^0_{10}\rangle - \Delta Q|\Lambda^1_0\rangle, \quad (4)$$

(!) In case of TS $\varphi_{\mu_1\ldots\mu_s}$ due to homogeneity in ghost number distribution

$$|\varphi_1\rangle \ = \ \varphi_{(\mu)_s}a^{\mu_1+}...a^{\mu_s+}|0\rangle + \eta_1^+\mathcal{P}_1^+D_{(\mu)_{s-2}}a^{\mu_1+}...a^{\mu_{s-2}+}|0\rangle,$$

$$|\varphi_2\rangle \ = \ \mathcal{P}_1^+C_{(\mu)_{s-1}}a^{\mu_1+}...a^{\mu_{s-1}+}|0\rangle, \quad |\Lambda_0\rangle \ = \mathcal{P}_1^+\Lambda_{(\mu)_{s-1}}a^{\mu_1+}...a^{\mu_{s-1}+}|0\rangle.$$

1.) Triplet formulation in terms of $\varphi_{(\mu)_s}, C_{(\mu)_{s-1}}, D_{(\mu)_{s-2}}$ reducible repr. $iso(1, d-1)$ (for $s = 1, 2$ -Bengtsson, 1984)
EoM (3) can be written schematically (with and) without oscillators as,

$$\partial^2\varphi_{(\mu)_s} - (\partial C)_{(\mu)_s} = 0, \quad (\partial\cdot\varphi)_{(\mu)_{s-1}} - (\partial D)_{(\mu)_{s-1}} = C_{(\mu)_{s-1}}, \quad (5)$$

$$\partial^2 D_{(\mu)_{s-2}} - (\partial\cdot C)_{(\mu)_{s-2}} = 0, \quad \delta\Big(\varphi, C, D\Big) \ = \Big(\partial, \partial^2, \partial\cdot\Big)\Lambda,$$

Motivations: SFT $\rightarrow$ reducible bosonic HS fields

$$l_0|\varphi\rangle_s - l_1^+|C\rangle_{s-1} = 0, \quad l_1|\varphi\rangle_s - l_1^+|D\rangle_{s-2} = |C\rangle_{s-1}, \tag{6}$$
$$l_0|D\rangle_{s-2} - l_1|C\rangle_{s-1} = 0$$
$$\delta\Big(|\varphi\rangle_s, |C\rangle_{s-1}, |D\rangle_{s-2}\Big) = \Big(l_1^+, l_0, l_1\Big)|\Lambda\rangle_{s-1} \tag{7}$$

System (5) or (6) is Lagrangian and is derived from the action

$$S(\Phi) = \int d\eta_{0s}\langle\Phi|Q|\Phi\rangle_s, \delta|\Phi\rangle_s = Q|\Lambda_0\rangle_s, \quad \text{for gh}(|\Phi\rangle, |\Lambda_0\rangle) = (0, -1) \tag{8}$$

(Francia, Sagnotti 2003) - massless reducible representations, $ISO(1, d-1)$ of HS fields with $(s, s-2, ..., 1/0)$
Imposing BRST-extended ($[Q, \mathcal{L}_{11}] = 0$) traceless constraints $\mathcal{L}_{11}$ on $|\Phi\rangle, |\Lambda_0\rangle$ $[Q, \mathcal{L}_{11}] = 0$:

$$\mathcal{L}_{11}\Big(|\Phi\rangle, |\Lambda_0\rangle\Big) = \Big(l_{11} + \eta_1 \mathrm{P}_1\Big)\Big(|\Phi\rangle, |\Lambda_0\rangle\Big) = (0, 0), \text{ for } l_{11} = 1/2a^\mu a_\mu. \tag{9}$$

2.) $\Longrightarrow$ to Fronsdal (1978) formulation literally with traceless $\Lambda_{(\mu)_{s-1}}$, and only surviving doubly traceless HS field $\varphi_{(\mu)_s}$.

3.) if we try to enlarge triplet F. up to minimal without higher derivatives GI LF incorporating Eq. (9) in so-called Unconstrained or quartet formulation, then addition of 1 compensator $\delta\alpha_{(\mu)_{s-3}} = (Tr\Lambda)_{(\mu)_{s-3}}$ with GI extension of 2 traceless constraints on fields

$$(Tr\varphi)_{(\mu)_{s-2}} - D_{(\mu)_{s-2}} + (\partial\alpha)_{(\mu)_{s-2}} = 0, \quad (TrD)_{(\mu)_{s-4}} + (\partial\cdot\alpha)_{(\mu)_{s-4}} = 0,$$
$$\Leftrightarrow \ l_{11}|\varphi\rangle_s - |D\rangle_{s-2} + l_1^+|\alpha\rangle_{s-3} = 0, \ \ l_{11}|D\rangle_{s-2} + l_1|\alpha\rangle_{s-3} = 0 \qquad (10)$$

makes joint system (5) and (10) by Lagrangian (with non-gauge HS tensors $\lambda_{1(\mu)_{s-2}}$, $\lambda_{2(\mu)_{s-4}}$ vanishing on shell determining by the EoM for the rest fields:

$$S(\Phi,\alpha,\lambda_1,\lambda_2) \ = \ S(\Phi) + \Big\{{}_{s-2}\langle\lambda_1|\Big(l_{11}|\varphi\rangle_s - |D\rangle_{s-2} + l_1^+|\alpha\rangle_{s-3}\Big) \qquad (11)$$
$$+{}_{s-4}\langle\lambda_2|\Big(l_{11}|D\rangle_{s-2} + l_1|\alpha\rangle_{s-3}\Big) + h.c.\Big\}),$$
$$\delta\Big(|\varphi\rangle_s, |C\rangle_{s-1}, |D\rangle_{s-2}, |\alpha\rangle_{s-3}, |\lambda_i\rangle_{s-2i}\Big) = \Big(l_1^+,\, l_0,\, l_1,\, l_{11}, 0\Big)|\Lambda\rangle_{s-1} \qquad (12)$$

(J.Buchbinder, A. Galajinskii, V.Krykhtin, 2007).

Motivations: SFT $\to$ bosonic HS fields

Summary 1: the triplet-like formalism for reducible HS theories exist for mixed-symmetric tensor fields (firstly, derived by Tsulaia & Sagnotti)
Summary 2: addition off-shell BRST extended traceless and Young symmetry constraints (on all the field vector and sequence of Gauge parameters) commuting with BRST operator leads to constrained GI LF of the form (8) for spin $(s_1, s_2 \dots, s_k)$ fields (Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev,Tipunin, 2008)
Summary 3: One may consider the problem from the beginning to find BRST-like action (8), so that the whole set of irreps Poincare group conditions follows as Lagrangian EoM, i.e.;

$$\partial^2 \varphi_{\mu\nu\dots} = 0, \quad \partial^\mu \varphi_{\mu\nu\dots} = 0, \quad \eta^{\mu\nu} \varphi_{\mu\nu\dots} = 0 \tag{13}$$

should be considered equally.
Sum. 3 is the essence of Unconstrained BRST-BFV approach for Lagrangian formulation construction for HS fields on $\mathbb{R}^{1,d-1}$ & AdS(d)
Summary 4: the same procedure of tensionless limit for the superstring theory leads to analogous triplet-like (Tsulaia & Sagnotti (2004)) , then to quartet-like formulations initially for reducible half-integer $ISO(1, d-1)$ reps and then irreps.

• We will call the any Lagrangian formulation (LF) for given HS field (or fields) by **Unconstrained LF for given HS field (or fields)**, if there are no (holonomic or not-holonomic) constraints on the field(s) and reducible gauge parameters which can not be produced from the Lagrangian action. In opposite case we will call the Lagrangian formulation (LF) for given HS field (or fields) by **Constrained LF for given HS field (or fields)**.

E.g. almost whole LFs for HS fields on $R^{1,d-1}$, AdS$_d$ obtained in frame-like formalism (E.Skvortsov, M.Vasiliev, Yu.Zinoviev, M.Grigoriev, D.Ponomarev) appear by Constrained LF due to algebraic constraints on field and g.parameters. The BRST and BRST-BV approaches developed in the recent papers by R.Metsaev in metric-like formalism appear by the Constrained LF as well.

The unconstrained LF as well were developed for (half-)integer HS fields on $R^{1.d-1}$ in Campoleoni A, Francia D, Mourad J and Sagnotti A, 2009, 2010 NPB

All the quantum results for HS theory are known from component (spin)-tensor formulations (e.g. for Fronsdal formulation of Bose fields on AdS(d) with 1-loop input for EA in H.P.Popova, K.V.Stepanyantz , arxiv:1511.06694, as well as the results of checking AdS/CFT correspondence). Because of reducible character of LF for HS theories (m = 0!) its L. quantization [Batalin, Vilkovisky, (1981,1983)].

General gauge theories:

$$S_0 = S_0(A),\ A^i; i = 1, 2, ..., n;\ \varepsilon(A^i) = \varepsilon_i,$$

$$\delta A^i = R^i_{\alpha_0}(A)\xi^{\alpha_0},\quad S_{0,i}R^i_{\alpha_0}(A) = 0,\quad R^i_{\alpha_0}(A)Z^{\alpha_0}_{\alpha_1}(A)\big|_{S_{0,i}=0} = 0, ...$$

$$\alpha_k = 1, 2, ..., m_k;\quad 0 < m_k < m_{k-1} < ... < m_0 < n,\quad \varepsilon(\xi^{\alpha_0}) = \varepsilon_{\alpha_0}, ...$$

$$\Phi^A = \Big\{A^i, C^{\alpha_0};\ \overbrace{\overline{C}^{\alpha_0}, B^{\alpha_0}, ...}^{nonminimal\ \ sector}\Big\},\ \varepsilon(\Phi^A) = \varepsilon_A,\ \Phi^*_A = \Big\{A^*_i, C^*_{\alpha_0};\ \overbrace{\overline{C}^*_{\alpha_0}, B^*_{\alpha_0}, ...}^{nonminimal\ \ sector}\Big\}$$

$$S = S(\Phi; \Phi^*),\quad \frac{1}{2}(S,\, S) = \imath\hbar\Delta S,\quad S\big|_{\Phi^*=0} = S_0(A)$$

Antibracket: $$(F, G) = F\frac{\overleftarrow{\delta}}{\delta\Phi^A}\frac{\overrightarrow{\delta}}{\delta\Phi^*_A}G - F\frac{\overleftarrow{\delta}}{\delta\Phi^*_A}\frac{\overrightarrow{\delta}}{\delta\Phi^A}G$$

Odd Laplacian: $$\Delta = (-1)^{\varepsilon_A}\frac{\overrightarrow{\delta}}{\delta\Phi^A}\frac{\overrightarrow{\delta}}{\delta\Phi^*_A},\ \Delta^2 = 0,\ \varepsilon(\Delta) = 1$$

Motivations: Lagrangian quantization rules

Extended action: $S_{ext}(\Phi;\Phi^*) = S\big(\Phi;\Phi^* + \psi\overleftarrow{\delta}/\delta\Phi\big)$

Gauge fixing Fermion: $\psi(\Phi) \overset{\text{irred.GTh YM}}{=} \overline{C}^{\alpha}\chi_{\alpha}(A,B),\ \Big(\chi = \overset{R_\xi-gauges}{\partial\cdot A + \xi/2B = 0}\Big)$

Quantum master equation: $\frac{1}{2}(S_{ext},\, S_{ext}) = \imath\hbar\Delta S_{ext}$

Generating functional of GF: $Z(J,\Phi^*) = \int D\Phi \exp\Big\{\frac{\imath}{\hbar}\big(S_{ext}(\Phi;\Phi^*) + J_A\Phi^A\big)\Big\},$

Ward identity for Z and EA: $J_A\frac{\overrightarrow{\delta}}{\delta\Phi^*_A}Z = 0,\ \ \big(\Gamma(\langle\Phi\rangle,\Phi^*),\Gamma(\langle\Phi\rangle,\Phi^*)\big) = 0,$

BRST symmetry: $\delta_B I(\Phi,\Phi^*) = 0,\ I = D\Phi\exp\Big\{\frac{\imath}{\hbar}\big(S_{ext} + J\Phi\big)\Big\},\ \delta_B\Phi^A = \mu\frac{\overrightarrow{\delta}}{\delta\Phi^*_A}S_{ext}.$

Problem: But there are not known (up to AR talk QFTG'2016 except for special reprs by R. Metsaev for QA) explicit quantization rules for BRST-BFV LFs of both free and interacting HS fields in terms of Fock space based objects above!

Reason: Not using operators $\overline{C}, \overline{P}, \lambda, \pi$, from nonminimal sector of BRST-BFV method to perform gauge-fixing via BFV-BV duality!

We develop within BRST-BFV approach for HS fields the Unconstrained LF, which after partial gauge-fixing may lost "Unconstrained" property.
A. R., PEPAN (2017) arXiv:1604.00620 (MAS $Y[s_1, s_2]$ on $R^{1,d-1}$ **developed earlier by** X.Bekaert, N.Boulanger, S, Cnockaert 2004;
A. R., NPB (2013) arXiv:1211.1273;
I. Buchbinder, A. R., NPB (2012) arXiv:1110.5044;
P.Moshin, A. R., JHEP 0710 (2007) 040 arXiv:0707.0386 ;
A. R., NPB 869 (2013) 523 arXiv:1211.1273;
and on AdS(d) space:
I.Buchbinder, V.Krykhtin, P.Lavrov NPB 2006 hep-th/0608005;
I.L. Buchbinder, V.A. Krykhtin, A. R., NPB B787 2007 hep-th/0703049;
C. Burdik, A. R.On representations of Higher Spin symmetry algebras for mixed-symmetry HS fields on AdS-spaces. Lagrangian formulation arXiv:1111.5516
Constrained BRST-BFV & BRST-BV on $R^{1,d-1}$ LF recently:
A. R., PEPAN (2018) arxiv:1803.05173;
A. R. Constrained BRST–BFV Lagrangian Formulations for Higher Spin Fields in Minkowski Spaces, arxiv:1803.04678

What are BRST-BFV and BRST-BV approaches to derive of (un)constrained Lagrangians for HS fields?

Within stringy-inspired BRST-BFV approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, J. Buchbinder, V. Krykhtin, A.R.) in opposite to direct problem of Gen. Ham. Quants of the Constrained dyn.systems the aim of the inverse problem consists in the CONSTRUCTION OF LF FOR HS FIELD WITH GIVEN (m, s)

$$\boxed{\begin{array}{c}\texttt{Irreps conditions}\\ \text{ISO(1,d-1), SO(2,d-1)}\end{array}} \xrightarrow{\texttt{SFT}} \boxed{\begin{array}{c}\texttt{(Super)algebra}\{o_I(x)\} : \mathcal{H}\\ [o_I, o_J\} = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0)\end{array}}$$

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$$\xrightarrow[\text{Burdik, Pashnev}]{\texttt{conversion}} \boxed{\begin{array}{c}O_I = o_I + o'_I : \mathcal{H}\bigotimes\mathcal{H}'\\ [O_I, O_J\} = F^K_{IJ}(o', O)O_K\end{array}}$$

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$$\xrightarrow[\text{Henneaux}]{\texttt{BFV}} \boxed{\begin{array}{c}\texttt{BRST operator for } \{O_I\}: Q'(x)\\ Q' = C^I O_I + \frac{1}{2} C^I C^J F^K_{IJ} \mathcal{P}_K + \text{more}\end{array}}$$

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$$\xrightarrow{\texttt{LF}} \boxed{\begin{array}{c}Q' = Q + (g_0 + h + \text{more})C_g + \ldots : Q^2 = 0\\ \texttt{mass-shell}: Q|\Psi\rangle = 0, \text{gh}(|\Psi\rangle) = 0 \Leftarrow \texttt{action}: S = \int d\eta_0 \langle\Psi|KQ|\Psi\rangle\\ \texttt{spin}: (g_0 + \text{more})(|\Psi\rangle, |\Lambda\rangle, \ldots) = -h(|\Psi\rangle, |\Lambda\rangle, \ldots)\\ \texttt{gauge transfs}: \delta|\Psi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \ldots\end{array}}$$

At 2-3rd steps the Stuckelberg and gauge fields are appeared automatically to obtain GI LF for basic field

What are BRST-BFV and BRST-BV approaches to derive of (un)constrained Lagrangians for HS fields?

For BRST-BV method the requirement $gh(\Psi_{(\mu)_s \dots}, \Lambda_{(\mu)_{s-1} \dots}) = 0$ for HS components in $|\Psi\rangle, |\Lambda\rangle, ..$ (used in BRST-BFV) is weaken:

$$gh(\Psi_{(\mu)_s \dots}, \Psi_{(\mu)_s \dots}) > 0 - \texttt{ ghost fields } gh(\Psi_{(\mu)_s \dots}, \Psi_{(\mu)_s \dots}) < 0 \texttt{ antifields}$$

$$\texttt{jointly in} |\Psi_{\mathrm{gen}}\rangle \Rightarrow \; S_{BV} = \int d\eta_0 \langle \Psi_{\mathrm{gen}} | KQ | \Psi_{\mathrm{gen}} \rangle = S(\Psi) + \text{"}more\text{"}$$

minimal Batalin-Vilkovisky action encoding (free) classical action $S(\Psi)$ and gauge algebra in "more".

For constrained case there are no 2nd-class constraints in the HS symmetry algebra, no conversion, $K = \hat{1}$, the spectrum of the components fields is smaller, there are off-shell (BRST-extended) constraints but the constrained dynamic will be equivalent to the unconstrained one!

The constrained BRST-BFV and BRST-BV approach for LF and minimal BV action was known only for integer spin cases(!) Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev,Tipunin, 2008, 2011); R.Metsaev 2012-2017

What problems to solve: unknown methods to derive LF and BV actions for fermionic HS fields?

- What can we state for (1) unconstrained BRST-BV method for integer HS fields?;
- What can we state for (2) constrained BRST-BFV and BRST-BV approaches for half-integer HS fields?;
- Does it possible to derive 1) the Fang-Fronsdal (1978) LF for massless spin-tensor $\Psi_{(\mu)_n}$;
  2) the triplet formulation for massless spin-tensor $\Psi_{(\mu)_n}$ (D. Francia, A. Sagnotti 2003,2005);
  3) the unconstrained quartet formulation as it was done for (half)-integer TS HS fields on $R^{1,d-1}$ (J.Buchbinder, A. Galajinskii, V.Krykhtin, 2007)?

1) the Fang-Fronsdal (1978) Lagrangian formulation for massless spin-tensor $\Psi_{(\mu)_n}$ $(s=(n+1/2))$: $\gamma^{\mu_1}\xi_{(\mu)_{n-1}}=0$, $\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\Psi_{(\mu)_n}=0$; $\delta\Psi^{(\mu)_n}=-\partial^{\{\mu_n}\xi^{(\mu)_{n-1}\}}$

$$\mathcal{S}_{c|(n)}(\Psi)=(-1)^n\int d^dx\overline{\Psi}^{(\nu)_n}\Big\{-\imath\gamma^\mu\partial_\mu\Psi_{(\nu)_n}+\frac{n(n-1)}{4}\eta_{\nu_{n-1}\nu_n}(\imath\gamma^\mu\partial_\mu)\eta^{\mu\rho}\Psi_{(\nu)_{n-2}\mu\rho}$$
$$-n\gamma_{\nu_n}(\imath\gamma^\mu\partial_\mu)\gamma^{\mu_n}\Psi_{(\nu)_{n-1}\mu_n}+n(\imath\partial_{\nu_n})\gamma^{\mu_n}\Psi_{(\nu)_{n-1}\mu_n}+n(\imath\partial^{\mu_n})\gamma_{\nu_n}\Psi_{(\nu)_{n-1}\mu_n}$$
$$-\frac{n(n-1)}{2}\Big(\gamma_{\nu_{n-1}}(\imath\partial_{\nu_n})\eta^{\mu_{n-1}\mu_n}\Psi_{(\nu)_{n-2}\mu_{n-1}\mu_n}+\eta_{\nu_{n-1}\nu_n}\gamma^{\mu_{n-1}}(\imath\partial^{\mu_n})\Psi_{(\nu)_{n-2}\mu_{n-1}\mu_n}\Big)\Big\}$$

What problems to solve: unknown methods to derive LF and BV actions for fermionic HS fields?

2) triplet formulation

$$\mathcal{S}_{c|(n)}(\Psi,\chi_1,\chi) = {}_n\langle\tilde{\Psi}|t_0|\Psi\rangle_n - {}_{n-2}\langle\tilde{\chi}|t_0|\chi\rangle_{n-2} + {}_{n-1}\langle\tilde{\chi}_1|\tilde{\gamma}t_0\tilde{\gamma}|\chi_1\rangle_{n-1}$$
$$- \Big({}_{n-1}\langle\tilde{\chi}_1|\tilde{\gamma}\{l_1|\Psi\rangle_n - l_1^+|\chi\rangle_n\} + h.c.\Big),$$

$$\delta\Big(|\Psi\rangle_n, |\chi_1\rangle_{n-2}, |\chi_1\rangle_{n-1}\Big) = \Big(l_1^+, l_1, \overset{=-\imath\gamma^\mu\partial_\mu}{\tilde{\gamma}t_0}\Big)|\xi\rangle_{n-1}$$

3) Unconstrained quartet formulation with compensator $|\varsigma\rangle_{n-2}$: $\delta|\varsigma\rangle_{n-2} = t_1|\xi\rangle_{n-1}$

$$\mathcal{S}_{(n)} = \mathcal{S}_{c|(n)}(\Psi,\chi_1,\chi) + \mathcal{S}_{\mathrm{add}|(n)}(\lambda)$$
$$\mathcal{S}_{\mathrm{add}|(n)}(\lambda) = {}_{n-1}\langle\tilde{\lambda}_1|\Big(t_1|\Psi\rangle_n - \tilde{\gamma}|\chi_1\rangle_{n-1} + l_1^+\tilde{\gamma}|\varsigma\rangle_{n-2}\Big) + {}_{n-2}\langle\tilde{\lambda}_2|\Big(|\chi\rangle_{n-2}$$
$$+\frac{1}{2}t_1\tilde{\gamma}|\chi_1\rangle_{n-1} + \frac{1}{2}t_0\tilde{\gamma}|\varsigma\rangle_{n-2}\Big) + {}_{n-3}\langle\tilde{\lambda}_3|\Big(t_1|\chi\rangle_{n-2} + l_1\tilde{\gamma}|\varsigma\rangle_{n-2}\Big) + h.c.,$$

2 ways to get Constrained BRST-BFV LF for (half-)integer HS fields on $R^{1,d-1}$: Reduction & self-consistency

Statement (A.R. arxiv:1803.04678)

The Constrained BRST-BFV Lagrangian Formulations for (half-)integer HS fields on $R^{1,d-1}$ subject to $Y(s_1, ..., s_k)$ are equivalent when constructed, first, by means of reduction from the Unconstrained BRST-BFV Lagrangian Formulation and, second, in self-consistent way by extracting from the (half-)integer HS symmetry algebra $\mathcal{A}^{(f)}(Y(k), \mathbb{R}^{1,d-1})$ the second-class (algebraic) constraint subsystem, from which the only half should be consistently imposed on the Hilbert space vectors.

Thus, for the fermionic $(m = 0)$ HS field the irreps $ISO(1, d-1)$:

$$\imath\gamma^{\mu}\partial_{\mu}\Psi_{(\mu^1)_{n_1},(\mu^2)_{n_2},...,(\mu^k)_{n_k}} = 0\,, \quad \gamma^{\mu^i_{l_i}}\Psi_{(\mu^1)_{n_1},(\mu^2)_{n_2},...,(\mu^k)_{n_k}} = 0\,, \tag{14}$$

$$\Psi_{(\mu^1)_{n_1},...,\{(\mu^i)_{n_i}\underbrace{,...,\mu^j_1...}\mu^j_{l_j}\}...\mu^j_{n_j},...(\mu^k)_{n_k}} = 0, \quad i < j, \ 1 \le l_j \le n_j, \tag{15}$$

correspond to the constraints on basic vector (spinor)

$$|\Psi\rangle = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{n_1}\cdots\sum_{n_k=0}^{n_{k-1}}\frac{\imath^{\sum_i n_i}}{n_1!\times ...\times n_k!}\Psi_{(\mu^1)_{n_1},(\mu^2)_{n_2},...,(\mu^k)_{n_k}}\prod_{i=1}^{k}\prod_{l_i=1}^{n_i}a_i^{+\mu^i_{l_i}}|0\rangle,$$

$$\tilde{t}_0|\Psi\rangle = \tilde{t}_i|\Psi\rangle = t_{rs}|\Psi\rangle = 0, \ \left(\tilde{t}_0\,, \tilde{t}_i\,, t_{rs}\right) = \left(-i\gamma^{\mu}\partial_{\mu}\,, \gamma_{\mu}a^{\mu}_i\,, a^+_{\mu r}a^{\mu}_s\right), \quad r < s,$$

Properly Constrained gauge-invariant Lagrangians for half-integer HS fields subject to $Y(s_1, ...s_k)$

$$\texttt{spin:} \quad g_0^i|\Psi\rangle = (n_i + \tfrac{d}{2})|\Psi\rangle\,, \text{ with } g_0^i = -\tfrac{1}{2}\{a_i^{\mu^i}, a^+_{i\mu^i}\}$$

The HS symmetry superalgebra (A.R. NPB 2013)

$$\mathcal{A}^f(Y(k),\mathbb{R}^{1,d-1}) = \{o_I\} = \{o_A; o_{\overline{a}}; o_{\underline{a}}; g_0^i\} \equiv \{t_0, l_0, l_i, l_i^+; t_i, t_{rs}, l_{ij} = \frac{1}{2}a_{\mu i}a_j^{\mu}; t_i^+, t_{rs}^+, l_{ij}^+; g_0^i\}$$

contains isometry subalgebra of the (differential) 1-class constraints o_A:

$\{o_A, o_B\} \sim o_C$, subsystem of the (algebraic) 2-nd class constraints $o_{\overline{a}}, o_{\underline{a}}$:

$\{o_{\overline{a}}, o_{\underline{a}}\} \overset{\Delta_{\overline{a}\underline{a}}\neq 0}{=} \Delta_{\overline{a}\underline{a}}(g_0^i) + (o_{\overline{a}}, o_{\underline{a}})$ which splits into subsystems of the 1-class constraints $o_{\overline{a}}$, $o_{\underline{a}}$ and g_0^i.

Here, the Grassmann-odd $\tilde{\gamma}^{\mu}$-matrices are introduced in $t_0, t_i, t_i^+ = \tilde{\gamma}^{\mu}(-i\partial_{\mu}, a_{i\mu}, a_{i\mu}^+)$ and H.C. "+".is determined with help of $\langle\ |\ \rangle$ odd scalar product in $\mathcal{H}$ differently for $d = 2N$ $(2N+1)$:

$\tilde{\gamma} = \Pi\gamma_{d+1}\delta_{D,2N} + \Pi\delta_{D,2N+1}$ for some $N \in \mathbf{N}$, $\Pi^2 = 1, \varepsilon(\Pi) = 1$,

$$\{\tilde{\gamma}^{\mu}, \tilde{\gamma}^{\nu}\} = 2\eta^{\mu\nu}, \quad \{\tilde{\gamma}^{\mu}, \tilde{\gamma}\} = 0([\tilde{\gamma}^{\mu}, \tilde{\gamma}] = 0), \quad \tilde{\gamma}^2 = -(-1)^d, \text{ so that } \gamma^{\mu} = \tilde{\gamma}^{\mu}\tilde{\gamma},$$

$$\langle\tilde{\Phi}|\Psi\rangle = \delta_{kl}\prod_{i=1}^{k}\delta_{n_i,p_i}\frac{(-1)^{\sum_j s_j}}{s_1!...s_k!}\int d^dx\,\Phi^+_{(\mu^1)_{n_1},...,(\mu^k)_{n_k}}\tilde{\gamma}_0\Psi^{(\mu^1)_{n_1},...,(\mu^k)_{n_k}}$$

$$\mathcal{A}^f(Y(k), \mathbb{R}^{1,d-1}) = \left(T^k \oplus T^{k*} \oplus [T^k, T^{k*}]\right) \not\ni osp(k|2k), \ \{T^{k(*)}; [\ ,\ ]\} = \{l_k^{(+)}; l_0 = -t_0^2\}. \quad (16)$$

The construction of the Hermitian constrained BRST operator $Q_c(o_A)$, spin operator $\sigma_c^i(g)$ and BRST-extended (algebraic) independent constraints $\{\mathcal{O}_a\} \subset \{\mathcal{O}_{\overline{a}} = o_{\overline{a}} + f_{\overline{a}}(\eta_i^{(+)}, P_i^{(+)})\}$ does not require the conversion procedure and realized in the Hilbert space $\mathcal{H}_c = \mathcal{H} \otimes \mathcal{H}_{gh}^{o_A}$ augmented by only Hamiltonian ghost oscillators to o_A is straightforward (as for Lie algebra and operator of number particles in $\mathcal{H}_c$)

$$[q_0, p_0] = \{\eta_0, \mathcal{P}_0\} = \imath, \ \{\eta_i, \mathcal{P}_j^+\} = \delta_{ij}, \ (p_0, \mathcal{P}_0)^+ = (p_0, -\mathcal{P}_0), \quad (17)$$

$$Q_c(o_A) = q_0 t_0 + \eta_0 l_0 + \eta_i^+ l^i + l^{i+}\eta_i + \imath\left(\textstyle\sum_i \eta_i^+ \eta^i - q_0^2\right)\mathcal{P}_0, \quad (18)$$

$$\sigma_c^i(g) = g_0^i + \eta_i^+ P_i - \eta_i P_i^+ : \ (\sigma_c^i(g))^+ = \sigma_c^i(g), \quad (19)$$

with except for $\mathcal{O}_a = o_a + f_a(\eta_i^{(+)}, P_i^{(+)})$ for $o_a = (t_i, t_{rs})$, because of $l_{ij} = \frac{1}{4}\{t_i, t_j\}$ which should found from the **generating equations** additional to $Q_c^2 = [Q_c, \sigma_c^i(g)] = 0$

$$[Q_c(o_A), \widehat{T}_i] = 0, \ [Q_c(o_A), \widehat{T}_{rs}] = 0, \ [\sigma_c^i(g), \widehat{T}_i] = \widehat{T}_i, \ [\sigma_c^i(g), \widehat{T}_{rs}] = (\delta_r^i - \delta_s^i)\widehat{T}_{rs} \quad (20)$$

providing compatibility of the L. dynamics for HS field with fixed spin subject to initial irreps with off-shell BRST-extended constraints.

Constrained GI Lagrangians for half-integer HS fields subject to $Y(s_1, ...s_k)$

The solution of (20):

$$\widehat{T}_i = t_i - \imath\eta_i p_0 - 2q_0\mathcal{P}_i, \quad \mathcal{L}_{lm} = l_{lm} + \frac{1}{2}\eta_{\{m}\mathcal{P}_{l\}}, \; l \leq m \quad (21)$$

$$\widehat{T}_{rs} = t_{rs} - \eta_r^+\mathcal{P}_s - \mathcal{P}_r^+\eta_s, \; r < s \quad (22)$$

From the spectral problem

$$Q_c|\chi_c\rangle = 0, \qquad \sigma_c^i|\chi_c\rangle = \left(n^i + \frac{d-2}{2}\right)|\chi_c\rangle, \qquad (\varepsilon, gh_H)\,(|\chi_c\rangle) = (1,0),$$

$$\delta|\chi_c\rangle = Q_c|\chi_c^1\rangle, \qquad \sigma_c^i|\chi_c^1\rangle = \left(n^i + \frac{d-2}{2}\right)|\chi_c^1\rangle, \qquad (\varepsilon, gh_H)\,(|\chi_c^1\rangle) = (0,-1),$$

$$\ldots\ldots \qquad \ldots\ldots\ldots\ldots \qquad \ldots\ldots$$

$$\delta|\chi_c^{s_c-1}\rangle = Q_c|\chi_c^{s_c}\rangle, \quad \sigma_c^i|\chi_c^{s_c}\rangle = \left(n^i + \frac{d-2}{2}\right)|\chi_c^{s_c}\rangle, \quad (\varepsilon, gh_H)\,(|\chi_c^{s_c}\rangle) = (s_c+1, -s_c),$$

$$\left(\widehat{T}_i, \widehat{T}_{rs}\right)|\chi_c^l\rangle = 0, \qquad l = 0, 1, ..., s_c.$$

$$|\chi_c\rangle = \sum_n q_0^{n_{b0}}\eta_0^{n_{f0}}\prod_{i,j}(\eta_i^+)^{n_{fi}}(\mathcal{P}_j^+)^{n_{pj}}|\Psi(a_i^+)^{n_{b0}n_{f0};(n)_{fi}(n)_{pj}}\rangle.$$

the solution is written as the second-order equations of motion and sequence of the reducible gauge transformations with off-shell constraints:

$$Q_c|\chi^0_c\rangle_{(n)_k} = 0, \delta\Big(|\chi^0_c\rangle_{(n)_k}, ..., |\chi^{s_c}_c\rangle_{(n)_k}\Big) = Q_c\Big(|\chi^1_c\rangle_{(n)_k}, \cdots, |\chi^{s_c+1}_c\rangle_{(n)_k}\Big), \delta|\chi^{s_c+1}_c\rangle_{(n)_k} = 0,$$

$$\Big(\widehat{T}_i, \widehat{T}_{rs}\Big)|\chi^l_c\rangle_{(n)_k} = 0, \quad l = 0, 1, ..., k, .$$

The corresponding BRST-like 2nd order constrained gauge-invariant action looks

$$\mathcal{S}^{(2)}_{c|(n)_k} = \int d\eta_{0\,(n)_k} \langle\tilde{\chi}^0_c|Q_c|\chi^0_c\rangle_{(n)_k}, \qquad (23)$$

Doing standard removing of second-order operator by partial gauge-fixing based on the decomposition in q_0, η_0

$$Q_c = q_0 t_0 + \eta_0 l_0 + -\imath(q_0^2 - \eta_i^+\eta_i)\mathcal{P}_0 + \Delta Q_c, \quad \Delta Q_c = \eta_i^+ l_i + \eta_i l_i^+,$$

$$|\chi^l_c\rangle = \sum_{e=0}^{k} q_0^e(|\chi^{l(e)}_{0|c}\rangle + \eta_0|\chi^{l(e)}_{1|c}\rangle), \; gh_H(|\chi^{l(e)}_{m|c}\rangle) = -(l+e+m+1), \; m = 0, 1$$

we find all components in powers of q_0, η_0 vector are removed except for 2 fields for each level

$$|\chi^s_c\rangle_{(n)_k} = |\chi^{s(0)}_{0|c}\rangle_{(n)_k} + q_0|\chi^{s(1)}_{0|c}\rangle_{(n)_k} - i\eta_0\tilde{t}_0|\chi^{s(1)}_{0|c}\rangle_{(n)_k},$$

Constrained GI Lagrangians for half-integer HS fields - $Y(s_1, ...s_k)$

Statement : (arxiv:1803.04678) The first-order constrained gauge-invariant LF for half-integer HS field, $\Psi_{(\mu^1)_{n_1},...,(\mu^k)_{n_k}}(x)$ with generalized spin $(s)_k = (n+\frac{1}{2})_k$, is determined by the action,

$$\mathcal{S}_{c|(n)_k} = \left({}_{(n)_k}\langle\tilde{\chi}^0_{0|c}| \;\; {}_{(n)_k}\langle\tilde{\chi}^1_{0|c}|\right) \begin{pmatrix} t_0 & \Delta Q_c \\ \Delta Q_c & t_0\eta_i^+\eta_i \end{pmatrix} \begin{pmatrix} |\chi^0_{0|c}\rangle_{(n)_k} \\ |\chi^1_{0|c}\rangle_{(n)_k} \end{pmatrix}$$

invariant with respect to the sequence of the reducible gauge transformations (for $s_c - 1 = (k-1)$-being by the the stage of reducibility):

$$\delta \begin{pmatrix} |\chi^{l(0)}_{0|c}\rangle_{(n)_k} \\ |\chi^{l(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} = \begin{pmatrix} \Delta Q_c & t_0\eta_i^+\eta_i \\ t_0 & \Delta Q_c \end{pmatrix} \begin{pmatrix} |\chi^{l+1(0)}_{0|c}\rangle_{(n)_k} \\ |\chi^{l+1(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix}, \; \delta \begin{pmatrix} |\chi^{k(0)}_{0|c}\rangle_{(n)_k} \\ |\chi^{k(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} = 0$$

(for $l = -1, 0, ..., k-1$ and $|\chi^{-1(m)}_{0|c}\rangle = 0$, $m = 0, 1$) with off-shell algebraically independent BRST-extended constraints imposed on the whole set of field and gauge parameters:

$$\widehat{T}_i\left(|\chi^{l(0)}_{0|c}\rangle_{(n)_k} + q_0|\chi^{l(1)}_{0|c}\rangle_{(n)_k}\right) = 0, \; \widehat{T}_{rs}|\chi^{l(m)}_{0|c}\rangle_{(n)_k} = 0 \; l = 0, 1, ..., k; \; m = 0, 1. \quad (24)$$

Note, the survived term $-i\eta_0\tilde{t}_0|\chi^{s(1)}_0\rangle_{(n)_k}$ does not give any contribution in (24) due to EoM. It is the first basic result of the work.

The crucial point that we established via analysis of respective Q, Q_c-complexes in $\mathcal{H}_{tot}$, $\mathcal{H}_c$: Unconstrained LF and Constrained LF are equivalent to the irreps (14),(15) !

From Constrained BRST-BFV LF to Constrained BRST-BV for (half)-integer HS fields. Case integer spin $Y(s_1)$

1) Introduce the **correspondence** among gauge parameter vectors, $|\chi_c^{l(e)}\rangle_{(n)_k}$, $l = 1, ..., k$, and respective ghost vectors, denoted as , $|C_c^{l(e)}\rangle_{(n)_k}$ with components spin-tensors from the minimal sector of field-antifield space, on which acts the Lagrangian ghost number, gh_L, distribution by the rule:

$$\Psi^{1(e_{b0})0_{f0};(n^1)_{fi}(n^1)_{pj}}_{(\nu^1)_{p^1_1}\cdots(\nu^k)_{p^1_k}} = C^{1(e_{b0})0_{f0};(n^1)_{fi}(n^1)_{pj}}_{(\nu^1)_{p^1_1}\cdots(\nu^k)_{p^1_k}}\mu_l \Longrightarrow |\chi_c^{1(e)}\rangle_{(n)_k} = |C_c^{1(e)}\rangle_{(n)_k}\mu_l,$$

$$\Psi^{2(e_{b0})0_{f0};(n^2)_{fi}(n^2)_{pj}}_{(\nu^1)_{p^2_1}\cdots(\nu^k)_{p^2_k}} = C^{2(e_{b0})0_{f0};(n^2)_{fi}(n^2)_{pj}}_{(\nu^1)_{p^2_1}\cdots(\nu^k)_{p^2_k}}\mu_2\mu_1 \Rightarrow |\chi_c^{2(e)}\rangle_{(n)_k} = |C_c^{2(e)}\rangle_{(n)_k}\mu_2\mu_1, \ ,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\Psi^{k(e_{b0})0_{f0};(n^k)_{fi}(n^k)_{pj}}_{(\nu^1)_{p^k_1}\cdots(\nu^k)_{p^k_k}} = C^{k(e_{b0})0_{f0};(n^k)_{fi}(n^k)_{pj}}_{(\nu^1)_{p^k_1}\cdots(\nu^k)_{p^k_k}}\mu_k...\mu_1 \Rightarrow |\chi_c^{k(e)}\rangle_{(n)_k} = |C_c^{k(e)}\rangle_{(n)_k}\mu_k...\mu_1,$$

with use of the Grassmann-odd scalars $\mu_1, ..., \mu_{k-1}$: $\{\mu_m, \mu_n\} = 0, \ m, n = 1, ..., k$, and $(\varepsilon, gh_{\rm H}, gh_{\rm L})\mu_k = (1, 0, -1)$: so that $\varepsilon\big(|C_c^{k(e)}\rangle_{(n)_k}\big) = \varepsilon\big(|\chi_c^{0(e)}\rangle_{(n)_k}\big)$.

2) Introduce so-called **total ghost number**, $gh_{\rm tot}$ as the sum of Lagrangian and Hamiltonian ghost numbers

$$gh_{\rm tot} = gh_H + gh_L, \qquad \big(gh_{\rm tot}, gh_H, gh_L\big)C^{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}_{(\nu^1)_{p^l_1}\cdots(\nu^k)_{p^l_k}} = (l-e, -e, l),$$

$$\big(gh_{\rm tot}, gh_H, gh_L\big)|C_c^{l(e)}\rangle_{(n)_k} = (-e, -l-e, l).$$

we may organize *generalized field vectors* for $e = 0, 1$

$$|\chi_c^{0(e)}\rangle_{(n)_k} \to |\chi_{\mathrm{gen}|c}^{0(e)}\rangle_{(n)_k} = |\chi_c^{0(e)}\rangle_{(n)_k} + \sum_{m=1}^{k} |C_c^{m(e)}\rangle_{(n)_k}, \ \big(\varepsilon, gh_{\mathrm{tot}}\big)|\chi_{\mathrm{gen}|c}^{0(e)}\rangle = (1, -e),$$

3) the spin-tensor antifields, $\Phi^*_{A_{\min}}$ with respective gradings:

$$\Phi^*_{A_{\min}} = \Big(\Psi^{*(\nu^1)_{p_1}\dots(\nu^k)_{p_k}}_{0(e_{b0})0_{f0};(n)_{fi}(n)_{pj}}, C^{*(\nu^1)_{p^l_1}\dots(\nu^k)_{p^l_k}}_{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}\Big), \ l = 1, ..., k,$$
$$\big(\varepsilon, gh_{\mathrm{tot}}, gh_H, gh_L\big)\Phi^*_{A_{\min}} = \big(1 + \varepsilon(\Phi^{A_{\min}}), -1 - gh_L(\Phi^{A_{\min}}), 0, -1 - gh_L(\Phi^{A_{\min}})\big),$$

as the external sources to the left, s, and right generators, $\overleftarrow{s}$, of *Lagrangian BRST-transformations* of the constrained classical, $\Psi^{0(e_{b0})0_{f0};(n)_{fi}(n)_{pj}}_{(\nu^1)_{p_1}\dots(\nu^k)_{p_k}}$, and ghost, $C^{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}_{(\nu^1)_{p^l_1}\dots(\nu^k)_{p^l_k}}$, $l = 1, ..., k$, fields parameterizing as $\Phi^{A_{\min}}$ the configuration space, $\mathcal{M}_{\min}$ in terms of Fock space vectors:

Constrained BRST-BV for half-integer HS fields

$$s\begin{pmatrix} |C^{l(0)}_c\rangle_{(n)_k} \\ |C^{l(1)}_c\rangle_{(n)_k} \end{pmatrix} = \begin{pmatrix} \Delta Q_c & t_0\eta^+_i\eta_i \\ t_0 & \Delta Q_c \end{pmatrix} \begin{pmatrix} |C^{l+1(0)}_{0|c}\rangle_{(n)_k} \\ |C^{l+1(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} \theta_{kl}, \ \ \theta_{kl} = 1, k > l$$

$$\Big({}_{(n)_k}\langle\tilde{C}^{l(0)}_c|, {}_{(n)_k}\langle\tilde{C}^{l(1)}_c|\Big)\overleftarrow{s} = \Big({}_{(n)_k}\langle\tilde{C}^{l+1(0)}_c|, {}_{(n)_k}\langle\tilde{C}^{l+1(1)}_c|\Big)\begin{pmatrix} \Delta Q_c & t_0 \\ t_0\eta^+_i\eta_i & \Delta Q_c \end{pmatrix}\theta_{kl},$$

$$\delta_B|C^{l(e)}_c\rangle_{(n)_k} \overset{def}{=} \mu s|C^{l(e)}_c\rangle_{(n)_k}, \ \delta_B\Big({}_{(n)_k}\langle\tilde{C}^{l(e)}_c|\Big) \overset{def}{=} {}_{(n)_k}\langle\tilde{C}^{l(e)}_c|\overleftarrow{s}\mu,$$

$$\big(\varepsilon, gh_{\mathrm{tot}}, gh_H, gh_L\big)\big[\mu,\ s\big] = \big[(1,-1,0,-1),(1,1,0,1)\big], \ \text{for } |C^{0(e)}_c\rangle \equiv |\chi^{0(e)}_c\rangle, l = 0, ..., k$$

The homogeneity in ε, gh_{tot} - gradings requires the following rule of organizing for any field vector $|C^{l(e)}_c\rangle_{(n)_k}$ its antifield vector $|C^{*l(e)}_c\rangle_{(n)_k}$ by changes in each monomial

$$\Big((\tilde{\gamma})^m; \eta^+_i, \mathcal{P}^+_j; C^{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}_{(\nu^1)_{p^l_1}\dots(\nu^k)_{p^l_k}}\Big) \mapsto \Big((\tilde{\gamma})^{1+m}; \mathcal{P}^+_i,\ \eta^+_j; C^{*(\nu^1)_{p^l_1}\dots(\nu^k)_{p^l_k}}_{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}\Big),$$

$$\text{e.g. for } \Big(\tilde{\gamma}|\Psi(a^+)\rangle_{(n)_k}, \mathcal{P}^+_i\tilde{\gamma}|\chi^{0(1)i}(a^+)\rangle_{(n)_k}\Big) \mapsto \Big(|\Psi^*(a^+)\rangle_{(n)_k}, \eta^+_i|\chi^{*0(1)i}(a^+)\rangle_{(n)_k}\Big)$$

$$\big(\varepsilon,\ gh_{\mathrm{tot}},\ gh_H,\ gh_L\big)\big[|\Psi^*(a^+)\rangle; |\chi^{*0(1)i}(a^+)\rangle\big] = (1,-1,0,-1).$$

Constrained BRST-BV for half-integer HS fields

Therefore, the vectors, $|C_c^{*l(e)}\rangle_{(n)_k}$, on $\mathcal{H}_c$ for each $e=0,1$ (and for $|C_c^{*0(e)}\rangle_{(n)_k} \equiv |\chi_c^{*0(e)}\rangle_{(n)_k}$) being homogeneous in $(\varepsilon, gh_{\mathrm{tot}})$-gradings can be combined into:

$$|\chi^{*0(e)}_{\mathrm{gen}|c}\rangle_{(n)_k} = |\chi_c^{*0(e)}\rangle_{(n)_k} + \sum_{m=1}^{k} |C_c^{*m(e)}\rangle_{(n)_k}, \ (\varepsilon, gh_{\mathrm{tot}})|\chi^{*0(e)}_{\mathrm{gen}|c}\rangle = (1, e-1), \ e=0,1,$$

which we will call as the *generalized antifield vectors*. As the result we come to the following minimal BV action

$$\begin{aligned} S_{c|(n)_k} \ = \ \mathcal{S}_{c|(n)_k} + \sum_{l=0}^{k-1}\Big\{ & \Big({}_{(n)_k}\langle\tilde{C}_c^{l+1(0)}|, {}_{(n)_k}\langle\tilde{C}_c^{l+1(1)}|\Big) \begin{pmatrix} \Delta Q_c & t_0 \\ t_0, \eta_i^+\eta_i & \Delta Q_c \end{pmatrix} \begin{pmatrix} |C^{*l(0)}_{0|c}\rangle_{(n)_k} \\ |C^{*l(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} \\ + \ & \Big({}_{(n)_k}\langle\tilde{C}_c^{*l(0)}|, {}_{(n)_k}\langle\tilde{C}_c^{*l(1)}|\Big) \begin{pmatrix} \Delta Q_c & t_0\eta_i^+\eta_i \\ t_0 & \Delta Q_c \end{pmatrix} \begin{pmatrix} |C^{l+1(0)}_{0|c}\rangle_{(n)_k} \\ |C^{l+1(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} \Big\}, \end{aligned}$$

we see, that it is invariant with respect to the **Lagrangian BRST-transformations** (25):

$$\delta_B S_{c|(n)_k} = 0 \ \text{ when } \ \delta_B|\chi^{*0(e)}_{\mathrm{gen}|c}\rangle_{(n)_k} = 0.$$

$$\delta_B |C_c^{l(e)}\rangle_{(n)_k} \stackrel{def}{=} \mu s |C_c^{l(e)}\rangle_{(n)_k}, \ \delta_B \Big({}_{(n)_k}\langle\tilde{C}_c^{l(e)}|\Big) \stackrel{def}{=} {}_{(n)_k}\langle\tilde{C}_c^{l(e)}|\overleftarrow{s}\mu,$$

subject to the off-shell BRST-extended constraints as for BRST-BFV for field vectors, whereas for antifield one should impose *antifield BRST-extended constraints* $\widehat{T}^*_i$, $\widehat{T}^*_{rs}$

$$\widehat{T}^*_i = t_i - 2\imath\eta_1 p_0 + q_0\mathcal{P}_1, \qquad \widehat{T}^*_{rs} = t_{rs} - \eta_r^+\mathcal{P}_s - \mathcal{P}_r^+\eta_s = \widehat{T}_{rs}, \ r<s.$$

Constrained BRST-BV for integer HS fields $Y(s_1)$

Example of constrained Fang-Fronsdal, triplet & unconstrained quartet minimal BRST-BV Lagrangians for TS field of spin $n+1/2$ considered in A.R. arxiv:1803.05173:
The Lagrangian, off-shell constraints, gauge transformations for free spin s HS field reads

$$S_{c|(s)}(\Phi) = \overrightarrow{\partial}\partial_{\eta_0\, s}\langle\chi^0_c|Q_c|\chi^0_c\rangle_s,\ \delta|\chi^0_c\rangle_s = Q_c|\chi^1_c\rangle_s,\ \widehat{L}_{11}|\chi^e_c\rangle_s = 0, e = 0,1$$

$$|\chi^0_c\rangle_s = |S_c\rangle_s + \eta_0|B_c\rangle_s = |\Phi(a^+)\rangle_s + \eta_0 P^+_1|\Phi_1(a^+)\rangle_{s-1} + \eta^+_1 P^+_1|\Phi_2(a^+)\rangle_{s-2}$$

$$= \Big(\frac{\imath^s}{\sqrt{s!}}\prod(a^+_\mu)\Phi^{(\mu)_s} + \eta_0 P^+_1\frac{\imath^{s-1}}{\sqrt{(s-1)!}}\prod(a^+_\mu)\Phi^{(\mu)_{s-1}}_1 + \eta_1 P^+_1\frac{\imath^{s-2}}{\sqrt{(s-2)!}}\prod(a^+_\mu)\Phi^{(\mu)_{s-2}}_2\Big)|0\rangle$$

$$|\chi^1_c\rangle_s = P^+_1|\xi_c(a^+)\rangle_{s-1} = P^+_1\frac{\imath^{s-1}}{\sqrt{(s-1)!}}a^{+\mu_1}\dots a^{+\mu_{s-1}}\xi_{(\mu)_{s-1}}|0\rangle$$

$$\mathbf{Q_c} = \eta_0 l_0 + \eta^+_1 l_1 + l^+_1\eta_1 + \imath\eta^+_1\eta_1\mathcal{P}_0 \ = \ \eta_0 l_0 + \Delta Q_c + \imath\eta^+_1\eta_1\mathcal{P}_0,$$

$$\widehat{\mathbf{L}}_{\mathbf{11}} = l_{11} + \eta_1\mathcal{P}_1, \qquad \widehat{\sigma}_{\mathbf{c}}(\mathbf{g}) \ = \ g_0 + \eta^+_1\mathcal{P}_1 - \eta_1\mathcal{P}^+_1$$

Resolution of the constraints leads to $\Rightarrow \xi_{(\mu)_{s-3}\mu\mu} = 0$

$$\Phi_1^{(\mu)_{s-3}\mu}{}_{\mu} = \Phi_2^{(\mu)_{s-4}\mu}{}_{\mu} = 0, \quad \frac{1}{2}\sqrt{s(s-1)}\Phi^{(\mu)_{s-2}\mu}{}_{\mu} = \Phi_2^{(\mu)_{s-2}} =$$

and the gauge transforms

$$\delta\big(\Phi^{(\mu)_s}, \Phi^{(\mu)_{s-1}}_1, \Phi^{(\mu)_{s-2}}_2\big) \ = \ \big(-\sqrt{\frac{1}{s}}\partial^{(\mu_s}\xi^{(\mu)_{s-1})}, \ \partial^2\xi^{(\mu)_{s-1}}, \ -\sqrt{s-1}\partial_{\mu_s}\xi^{(\mu)_{s-1}}\big)$$

Constrained BRST-BV for integer HS fields $Y(s_1)$

The action in η_0- (triplet), ghost- and oscillator- independent (Fronsdal) forms

$$\mathcal{S}_{C|(s)} = \left({}_s\langle S_c\big|{}_s\langle B_c\big|\right)\begin{pmatrix} l_0 & -\Delta Q_c \\ -\Delta Q_c & \eta_1^+\eta_1 \end{pmatrix}\begin{pmatrix} \big|S_c\rangle_s \\ \big|B_c\rangle_s \end{pmatrix}, \text{ for } \Delta Q_c = \eta_1 l_1^+ + \eta_1^+ l_1,$$

$$\mathcal{S}_{C|(s)} = \left({}_s\langle\Phi\big|{}_{s-2}\langle\Phi_2\big|{}_{s-1}\langle\Phi_1\big|\right)\begin{pmatrix} l_0 & 0 & -l_1^+ \\ 0 & -l_0 & l_1 \\ -l_1 & l_1^+ & 1 \end{pmatrix}\begin{pmatrix} \big|\Phi\rangle_s \\ \big|\Phi_2\rangle_{s-2} \\ \big|\Phi_1\rangle_{s-1} \end{pmatrix},$$

$$\delta\begin{pmatrix} \big|S_c\rangle_s \\ \big|B_c\rangle_s \end{pmatrix} = \begin{pmatrix} \Delta Q_c \\ l_0 \end{pmatrix}\big|S^1\rangle_s \Longleftrightarrow \delta\left(\big|\Phi\rangle_s, \big|\Phi_2\rangle_{s-2}, \big|\Phi_1\rangle_{s-1}\right) = \left(l_1^+, l_1, l_0\right)|\xi_c\rangle_{s-1}$$

for $||S^1\rangle_s \equiv ||\chi^1_c\rangle_s$. and to the Fang-Fronsdal LF and etc....

$$\mathcal{S}_{F|(s)}\left(|\Phi\rangle\right) = {}_s\langle\Phi\big|\left(l_0 - l_1^+l_1 - (l_1^+)^2 l_{11} - l_{11}^+ l_1^2 - l_{11}^+(l_0 + l_1 l_1^+)l_{11}\right)\big|\Phi\rangle_s,$$

$$\delta\big|\Phi\rangle_s = l_1^+|\xi_c\rangle_{s-1} \quad \text{and} \quad l_{11}\left(l_{11}|\Phi\rangle, |\xi_c\rangle\right) = (0,0),$$

$$\mathcal{S}_{F|(s)}(\Phi) = (-1)^s \int d^d x \Big\{ \Phi_{(\mu)_s}\big(\partial^2 \Phi^{(\mu)_s} - s\partial^{\mu_s}\partial_\nu \Phi^{(\mu)_{s-1}\nu} + s(s-1)\partial^{\mu_{s-1}}\partial^{\mu_s}\Phi^{(\mu)_{s-2}}$$

$$-\frac{1}{2}s(s-1)\Phi_{(\mu)_{s-2}\mu}{}^{\mu}\big(\partial^2\Phi^{(\mu)_{s-2}\nu}{}_\nu + \frac{1}{2}(s-2)\partial^{\mu_{s-2}}\partial^{\mu}\Phi^{(\mu)_{s-3}}{}_{\mu\nu}{}^{\nu}\big)\Big\},$$

Constrained BRST-BV for integer HS fields $Y(s_1)$

Following, to W.Siegel, 1985, but in another way, we extend the configuration space of BRST-BFV LF formulation (triplet or Fronsdal) by another HS tensors following to half-integer HS case. Thus, *generalized vector* instead of $|\chi_c\rangle$ looks as

$$|\chi_{g|c}\rangle = \sum_n \prod (b_1^+)^{n_1} (\eta_0)^{n_{f0}} \prod (\eta_1^+)^{n_{f1}} (\mathcal{P}_1^+)^{n_{p1}} |\Phi(a^+)^{n_{f0}n_{f1}n_{p1}}_{g|c}\rangle \quad (25)$$

$$= \sum_n \frac{\imath^n}{\sqrt{n!}} \prod (\eta_0)^{n_{f0}} \prod (\eta_1^+)^{n_{f1}} (\mathcal{P}_1^+)^{n_{p1}} \Phi^{n_{f0}n_{f1}n_{p1}}_{g|c(\mu)_n} a^{\mu_1+} \dots a^{\mu_n+} |0\rangle,$$

where $\Phi(a^+)^{n_{f0}n_{f1}n_{p1}}_{g|c} = \Phi(a^+)^{n_{f0}n_{f1}n_{p1}}_{c}$ for $gh_L(\Phi(a^+)^{n_{f0}n_{f1}n_{p1}}_{g|c}) = 0$. From the same spectral problem for Q_c-complex but in BRST-BV we get spin and modified total ghost numbers distributions which leads for incorporating in $|\chi^0_{g|c}\rangle_s$: $gh_{tot}(|\chi^0_{g|c}\rangle) = 0$ whole set of (anti)fields of minimal BV sector for given BRST-BFV LF $(\Phi^A_{\min}, \Phi^*_{A\,\min})$:

$$\mathcal{M}_{\min} = \{\Phi_k^{(\mu)_{s-k}}, C^{(\mu)_{s-1}}\}(x) = \{\Phi^A_{\min}\},\ \Pi T^*_{\Phi_{\min}}\mathcal{M}_{\min} = \{\Phi^*_{k(\mu)_{s-k}}, C^*_{(\mu)_{s-1}}\}(x),$$

	a^+	C^I	$\mathcal{P}_I$	$\Phi_k^{(\mu)_{s-k}}$	$C^{(\mu)_{s-1}}$	$\Phi^*_{k(\mu)_{s-k}}$	$C^*_{(\mu)_{s-1}}$
gh_H	0	1	-1	0	0	0	0
gh_L	0	0	0	0	1	-1	-2
$gh_{\rm tot}$	0	1	-1	0	1	-1	-2
ε	0	1	1	0	1	1	0

Table: Ghost numbers and Grassman parity distributions.

$$|\chi^0_{\mathrm{g}|c}\rangle_s = |\chi^0_c\rangle_s + \mathcal{P}^+_1 C_{s-1}(a^+)|0\rangle + |\chi^{0*}_c\rangle_s - \eta_0\eta^+_1 C^*_{s-1}(a^+)|0\rangle.$$
$$|\chi^{0*}_c\rangle_s = |S^*_c\rangle_s + \eta_0|B^*_c\rangle_s = \eta^+_1|\Phi^*_1(a^+)\rangle_{s-1} + \eta_0|\Phi^*(a^+)\rangle_s + \eta_0\mathcal{P}^+_1\eta^+_1|\Phi^*_2(a^+)\rangle_{s-2}$$

The *classical antifield* vector $|\chi^{0*}_c\rangle_s$ is naturally constructed from the *classical field* vector $|\chi^0_c\rangle_s$ by changing of

$$\eta_0^{n_{f0}}\eta_1^{+n_{f1}}\mathcal{P}_1^{+n_{p1}}|\Phi(a^+)_c^{n_{f0}n_{f1}n_{p1}}\rangle \to \eta_0^{(n_{f0}+1)\mathrm{mod}2}\mathcal{P}_1^{+n_{f1}}\eta_1^{+n_{p1}}|\Phi^*(a^+)_c^{n_{f0}n_{f1}n_{p1}}\rangle.$$

respecting the spin and ghost numbers relations. The minimal BV action in η_0-independent, in component triplet and Fronsdal-like forms reads as follows,

$$S_{C|(s)} = \int d\eta_0\, {}_s\langle\chi^0_{\mathrm{g}|c}|Q_c|\chi^0_{\mathrm{g}|c}\rangle_s = \mathcal{S}_{C|(s)} - {}_s\langle\chi^{0*}_c|Q_c|C\rangle_s + {}_s\langle C|Q_c|\chi^{0*}_c\rangle_s\Big\}$$
$$= \mathcal{S}_{C|(s)} + \left({}_s - \left(\langle S^*_c|, \langle B^*_c|\right)\begin{pmatrix} l_0 & -\Delta Q_c \\ -\Delta Q_c & \eta^+_1\eta_1\end{pmatrix}\begin{pmatrix}|C\rangle_s, \\ 0\end{pmatrix} + h.c.\right)$$
$$= \mathcal{S}_{C|(s)} + \left(\left(-{}_{s-1}\langle\Phi^*_1\big|l_0 + {}_s\langle\Phi^*_0\big|l^+_1 + {}_{s-2}\langle\Phi^*_2\big|l_1\right) C_{s-1}(a^+)|0\rangle + h.c.\right)$$
$$\mathcal{S}_{Fs}(\Phi_0,\Phi^*_0) = \mathcal{S}_{Fs}(\Phi) + (-1)^s\int d^dx\Phi^*_{0|(\mu)_s}\left(-\frac{1}{\sqrt{s}}\partial^{\{\mu_s}\right)C^{(\mu\})_{s-1}},$$

The functional $S_{C|(s)}$ is invariant w.r.t. the Lagrangian BRST-transformations in

BRST-BV approach for Lagrangian formulations: minimal BV action, antibracket, Δ_H on Fock-space $\mathcal{H}_{tot}$

$$\delta_B|\chi^0_c(x)\rangle_s = \mu\frac{\overrightarrow{\delta}}{\delta_s\langle\chi^*_c(x)|}S_{C|(s)} = Q_c|C(x)\rangle_s\mu \overset{|\chi^1_c\rangle\equiv|C\rangle\mu}{=} Q_c|\chi^1_c(x)\rangle_s,$$

$$\delta_B|C(x)\rangle_s = \frac{\overrightarrow{\delta}}{\delta_s\langle C^*(x)|}S_{C|(s)}\mu = 0, \quad \varepsilon\left(\frac{\overrightarrow{\delta}}{\delta_s\langle\chi^*_r(x)|}, \frac{\overrightarrow{\delta}}{\delta_s\langle C^*(x)|}\right) = 1,$$

or, equivalently, in terms of *new BRST generator*, s_0:

$$\delta_B\left[|\chi^0_c(x)\rangle_s, |C(x)\rangle_s\right] = \mu s_0\left[|\chi^0_c(x)\rangle_s, |C(x)\rangle_s\right] = \mu\left[Q_c, 0\right]|C(x)\rangle_s.$$

Now, we may formulate BRST transforms & *master equation* in terms of an odd x^μ-local Poisson antibracket $(\cdot,\cdot)^{\mathcal{H}}_s$ acting on the functionals given on the Hilbert space $\mathcal{H}^{\rm g}_{tot}$ $(gh_L(\mathcal{H}^{\rm g}_{tot}) \neq 0)$:

$$\left(S_{C|(s)}, S_{C|(s)}\right)^{\mathcal{H}}_s = -2\int d\eta_0\, d^dx \left\{\frac{S_{C|(s)}\overleftarrow{\delta}}{\delta|\chi_c(x)\rangle_s}\frac{\overrightarrow{\delta}S_{C|(s)}}{\delta_s\langle\chi^*_c(x)|} - \frac{S_{C|(s)}\overleftarrow{\delta}}{\delta|C(x)\rangle_s}\frac{\overrightarrow{\delta}S_{C|(s)}}{\delta_s\langle C^*(x)|}\right.$$
$$\left. + \frac{S_{C|(s)}\overleftarrow{\delta}}{\delta|\chi^*_c(x)\rangle_s}\frac{\overrightarrow{\delta}S_{C|(s)}}{\delta_s\langle\chi_c(x)|} - \frac{S_{C|(s)}\overleftarrow{\delta}}{\delta|C^*(x)\rangle_s}\frac{\overrightarrow{\delta}S_{C|(s)}}{\delta_s\langle C(x)|}\right\} = 0.$$

The antibracket is naturally determined on the space of quadratic functionals on $\mathcal{H}^{\rm g}_{tot}$

$$F_0(\chi,\chi^*) = \int d\eta_0\; {}_s\langle\chi_{{\rm g}|r}|E_F|\chi_{{\rm g}|r}\rangle_s, \text{ for } E_F = E_F(\mathcal{C},\mathcal{P},O_I),$$

The antibracket is easily continued to act on a space of arbitrary regular functionals $F, G \in C[\Pi T^*\mathcal{M}_{\min}]$ realized via tensor products, $(\mathcal{H}^{g}_{tot})^{\bigotimes p}$, $p = 2, \ldots$:

$$F(\chi, \chi^*) = \int d\eta_0 \Big\{ \langle \chi_{g|r} | E_F | \chi_{g|r} \rangle + \sum_{p=3}^{M} \Big[\otimes_{k=1}^{p} \left(\langle \chi_{g|r} | \right)^k \big| V_F^{(p)} \rangle + \langle W_F^{(p)} \big| \otimes_{k=1}^{p} \left(| \chi_{g|r} \rangle \right)^k \Big] \Big\},$$

$$\text{for } \big| V_F^{(p)} \rangle = V_F^{(p)}(\mathcal{C}, \mathcal{P}, O_I, a^+) \otimes_p |0\rangle, \quad \langle W_F^{(p)} \big| = \left(\otimes_p \langle 0 | \right) W_F^{(p)}(\mathcal{C}, \mathcal{P}, O_I, a).$$

For real-valued F the kernel E_F: $E_F = E_F^+$. In addition, there exists an odd nilpotent second order operator $\Delta_s^{\mathcal{H}}$:

$$\Delta^{\mathcal{H}}_s = \int d\eta_0\, d^d x\, \mathrm{tr}_{\mathcal{H}} \Big\{ \frac{\overrightarrow{\delta}}{\delta |\chi_c(x)\rangle_s} \frac{\overrightarrow{\delta}}{\delta_s \langle (\chi^*_c)^\star(x)|} - \frac{\overrightarrow{\delta}}{\delta |\chi^*_c(x)\rangle_s} \frac{\overrightarrow{\delta}}{\delta_s \langle \chi^*_c(x)|} + \frac{\overrightarrow{\delta}}{\delta |C^*(x)\rangle_s} \frac{\overrightarrow{\delta}}{\delta_s \langle C^\star(x)|} - \frac{\overrightarrow{\delta}}{\delta |C(x)\rangle_s} \frac{\overrightarrow{\delta}}{\delta_s \langle C^*|} \Big\},$$

[with trace operation $\mathrm{tr}_{\mathcal{H}}$ acting only on oscillators $a^{(+)}_\mu$, $\eta^{(+)}_1$, $\mathcal{P}^{(+)}_1$ in the unity $\hat{1}_s$ decomposition,] which deviation to satisfy Leibnitz rule when it differentiates the products of 2 functionals F, G generates the antibracket:

$$\Delta^{\mathcal{H}}_s(FG) - \left\{ (\Delta^{\mathcal{H}}_s F) G + (-1)^{\varepsilon(F)} F \Delta^{\mathcal{H}}_s G \right\} = (-1)^{\varepsilon(F)} (F, G)^{\mathcal{H}}_s ,$$

To establish relation to the conventional antibracket, Δ we should has the explicit definition of the functional derivatives (like in $N=1$ superfield BRST quantization (P.Lavrov, P.Moshin, A.R. 1995)),e.g. :

$$\left(\frac{F\overleftarrow{\delta}}{\delta|\chi_c^{(*)}\rangle_s};\frac{\overrightarrow{\delta}G}{\delta_s\langle\chi_c^{(*)}|}\right)=\left(\eta_0\frac{\overline{F}\overleftarrow{\delta}_{\eta_0}}{\delta|S_c^{(*)}\rangle_s}+(-1)^{\varepsilon(F)+1}\frac{\overline{F}\overleftarrow{\delta}_{\eta_0}}{\delta|B_c^{(*)}\rangle_s};\eta_0\frac{\overrightarrow{\delta}_{\eta_0}\overline{G}}{\delta_s\langle S_c^{(*)}|}-\frac{\overrightarrow{\delta}_{\eta_0}\overline{G}}{\delta_s\langle B_c^{(*)}|}\right)$$

$$\frac{\overrightarrow{\delta}_{\eta_0}\overline{G}}{\delta_s\langle B_c^{*}|}=\left(\frac{\imath^s}{\sqrt{s!}}\frac{\overrightarrow{\delta}}{\delta\Phi^*_{0|(\mu)_s}}a^+_{\mu_1}...a^+_{\mu_s}\hat{P}_0+\frac{\imath^{s-2}}{\sqrt{(s-2)!}}\eta_1^+\mathcal{P}_1^+\frac{\overrightarrow{\delta}}{\delta\Phi^*_{(\mu)_{s-2}}}a^+_{\mu_1}...a^+_{\mu_{s-2}}\hat{P}_1\right)\frac{\overrightarrow{\delta}}{\delta\langle 0|}\overline{G}$$

$$\left[\widetilde{F}\left(\Phi_{\min},\Phi^*_{\min}\right),\widetilde{G}\left(\Phi_{\min},\Phi^*_{\min}\right)\right]=\left[\overline{F}(S_c^{(*)},B_c^{(*)}),\overline{G}(S_c^{(*)},B_c^{(*)})\right]=\left[F(\chi_{g|c}),G(\chi_{g|c})\right]$$

As the result the relations hold

$$\left(\overline{F},\overline{G}\right)^{\mathcal{H}}_s=2(-1)^s\left(\widetilde{F},\widetilde{G}\right)^{\min}_s,\quad \Delta_s^{\mathcal{H}}\overline{F}=2(-1)^s\Delta_s^{\min}\widetilde{F}$$

here the $\mathbf{R}$-valued (anti)field HS tensors, whereas for the $\mathbf{C}$-valued ones there are no factor "2".

On consistent interaction from minimal BRST-BV Lagrangian

BRST-BV presentation permits one to serve consistency when derive interaction vertexes (see R. Metsaev in metric-like form). For the cubic case the problem maybe solved by deforming both $\mathcal{S}_{C|(s)}(\Phi)$ and the gauge transforms within the gauge theory with the same field $\Phi_{(\mu)_s}$ by self-interaction terms $\mathcal{S}_{1|(s)}(\Phi)$ and $S_{1g|(s)}(\Phi, \Phi^*, C)$:

$$S_{1|(s)}(\Phi, \Phi^*, C, C^*) = S_{C|(s)}(\Phi, \Phi^*, C) + \mathcal{S}_{1|(s)}(\Phi) + S_{1g|(s)}(\Phi, ...) + S_{2g|(s)}(\Phi, \Phi^*, C, C^*),$$

$$\mathcal{S}_{1|(s)}(\Phi) = {}_s\langle\Phi|\otimes \Big({}_s\langle V(l_1, l_{11})|\Phi\rangle_s \otimes |\Phi\rangle_s + {}_s\langle\Phi|V^+\rangle_s \otimes |\Phi\rangle_s\Big),$$

$$S_{1g|(s)} = {}_s\langle\Phi^*| \otimes {}_{s-1}\langle V_1(l_1, l_{11})|\Phi\rangle_s \otimes |C\rangle_{s-1} + {}_{s-1}\langle C| \otimes {}_s\langle\Phi|V_1^+\rangle_{s-1} \otimes |\Phi^*\rangle_s,$$

$$S_{2g|(s)} = \frac{1}{2}\Big\{{}_{s-1}\langle C^*| \otimes {}_{s-1}\langle F(l_1, l_{11})|C\rangle_{s-1} \otimes |C\rangle_{s-1} + {}_{s-1}\langle C| \otimes {}_{s-1}\langle C|F^+\rangle_{s-1} \otimes |C^*\rangle_{s-1}\Big\}$$

where the last term is needed to have closed deformed algebra of non-Abelian gauge transforms determined with , ${}_{s-1}\langle V_1(l_1, l_{11})|$:

$$\delta_{[1]}|\Phi\rangle_s = (\delta_0 + \delta_1)|\Phi\rangle_s = \Big(l_1^+ + {}_{s-1}\langle V_1(l_1, l_{11})|\Phi\rangle_s \otimes \Big)|\xi\rangle_{s-1},$$

$${}_{s-1}\langle V_1| = \sum_{k=0}^{\left[\frac{s-1}{2}\right]} b_k(l_{11})^{\left[\frac{s-1}{2}\right]-k}(gl_1)^{s-2\left[\frac{s-1}{2}\right]+2k-1},$$

with dimensionless coupling constant g, unknowns numbers b_k, $k = 0, ..., \left[\frac{s-1}{2}\right]$, $[x]$ - integer part of the real x. The structure of the vertex in $\mathcal{S}_{1|s}(\Phi)$ determined by the analogous as for V_1 general local ansatc for unknowns ${}_s\langle V(l_1, l_{11})|$, ${}_{s-1}\langle F$ to be determined from the validity master equation for the action $S_{1|s}(\Phi, \Phi^*, C, C^*)$ with accuracy up to the second order in $|C\rangle_{s-1}$

Extension of minimal BRST-BFV (BRST-BV) to non-minimal BRST-BFV (BRST-BV) for HS fields

that guarantees the closedness of commutator of deformed gauge transformations up to the 2nd powers in Φ:

$$\left(S_{1|(s)},\, S_{1|(s)}\right)^{\mathcal{H}}_{s} = o(C^2) \Rightarrow \left[\delta_{[1],\xi_1},\, \delta_{[1],\xi_2}\right]|\Phi\rangle_s = \delta_{[1],\xi_3}|\Phi\rangle_s + o(\Phi^2),\ \xi_3 = \xi_3(\xi_1,\xi_2).$$

Skip the way of covariant GF procedure by antighost and Nakanishi-Lautrup field vectors without changing $\mathcal{H}_{tot}$ structure (as R.Metsaev, 2015) let's extend both BRST-BFV and BRST-BV method for HS fields by introduction of nonminimal antighost and Lagrangian multipliers oscillators ($N_{nmin} = 4[o_I]$, $N_{nmin}(Y(s_1)) = 8$) to write nonminimal BRST operator, e.g for constrained case as $Q'_{c|tot}$, $Q_{c|tot}$:

$$\varepsilon(\overline{C}^I, \overline{\mathcal{P}}_J, \lambda_I, \pi^J) = (\varepsilon_I + 1, \varepsilon_J + 1, \varepsilon_I, \varepsilon_J);\ gh_H(\overline{C}^I, \overline{\mathcal{P}}_J, \lambda_I, \pi^J) = (-1, 1, 0, 0);$$
$$[\overline{C}^I,\, \overline{\mathcal{P}}_J\} = [\lambda_I,\, \pi^J\} = \delta^I_J.$$

So that the total BRST operator, Q' to construct the unitarizing Hamiltonian for (topological) dyn. system (Batalin, Fradkin, Vilkovisky, 1977) is determined:

$$Q'_{\rm tot} = Q' + \overline{\mathcal{P}}_I\pi^I = \mathcal{C}^I\big(O_I + \frac{1}{2}\mathcal{C}^J f^K_{JI}\mathcal{P}_K\big) + \overline{\mathcal{P}}_I\pi^I,$$

$$Q'_{c|\rm tot} = Q'_c + \overline{\mathcal{P}}^+_1\pi_1 + \overline{\mathcal{P}}_1\pi^+_1 = Q_r + \eta_g\big(g_0 + \eta^+_1\mathcal{P}_1 - \eta_1\mathcal{P}^+_1\big) + \overline{\mathcal{P}}^+_1\pi_1 + \overline{\mathcal{P}}_1\pi^+_1,$$

$$Q_{c|\rm tot} = Q_c + \overline{\mathcal{P}}^+_1\pi_1 + \overline{\mathcal{P}}_1\pi^+_1 = \eta_0 l_0 + \eta^+_1 l_1 + \eta_1 l^+_1 + \imath\eta^+_1\eta_1\mathcal{P}_0 + \overline{\mathcal{P}}^+_1\pi_1 + \overline{\mathcal{P}}_1\pi^+_1,$$

Extension of minimal BRST-BFV (BRST-BV) to non-minimal BRST-BFV (BRST-BV) for HS fields

Again extending minimal BRST-BV method to non-minimal the new non-minimal BRST-BV action for total generalized field vector $|\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\rangle_s \in \mathcal{H}_{\mathrm{tot}} \otimes \mathcal{H}_{\mathrm{nmin}}$:

$$S_0(\chi^{\mathrm{tot}0}_{g|c}) \; = \; \int d\eta_0 \; {}_s\langle\chi^{\mathrm{tot}0}_{g|c}\big|Q_{c|\mathrm{tot}}\big|\chi^{\mathrm{tot}0}_{g|c}\rangle_s,, \quad \mathcal{L}_{11}\big|\chi^{\mathrm{tot}0}_{g|r}\rangle_s = 0.$$

$$\begin{aligned} |\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\rangle_s \; &= \; |\chi^0_c\rangle_s + |C(\mathcal{P}^+_1, a^+)\rangle_s + |\chi^{0*}_c\rangle_s + |C^*(\eta_0, \eta^+_1, a^+)\rangle_s \\ &+ .\overline{\mathcal{P}}^+_1\big|\overline{C}(a^+)\rangle_{s-1} + \lambda^+_1\big|b(a^+)\rangle_{s-1} + \eta_0\overline{\eta}^+_1\big|\overline{C}^*(a^+)\rangle_{s-1} + \eta_0\pi^+_1\big|b^*(a^+)\rangle_{s-1} \\ &\equiv \; |\chi^0_{\mathrm{g}|\mathrm{c}}\rangle_s + \big|\overline{C}(\overline{\mathcal{P}}^+_1, a^+)\rangle_s + \big|b(\lambda^+_1, a^+)\rangle_s + \big|\overline{C}^*(\eta_0, \overline{\eta}^+_1, a^+)\rangle_s + \big|b^*(\eta_0, \pi^+_1, a^+)\rangle_s, \end{aligned}$$

$(\varepsilon, gh_{tot})|\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\rangle = (0,0)$**!** The action $S_0(\chi^{\mathrm{tot}0}_{g|c})$ is nothing else as BV action S_{ext}(!) .The (quantum) master equation remain the same but now rewritten in terms of the antibracket $(\bullet, \bullet)^{\mathcal{H}}_{\mathrm{tot}|s}$ and operator Δ_s given with help of $|\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\rangle$ and its dual. To determine QUANTUM ACTION S_{ext} = introduce Quadratic gauge fermion Functional $\Psi(\chi^{\mathrm{tot}0}_{g|c})$ corresponding to $R_{\xi,\beta}$-gauges with help of x-local kernel being **BRST-BFV gauge-fermion**:

$$\boldsymbol{\Psi}\left(\chi^{\mathrm{tot}\mathbf{0}}_{\mathrm{g}|\mathbf{c}}\right) = \int \mathbf{d}\eta_0 \left\{ {}_{\mathbf{s}}\langle\chi^{\mathrm{tot}\mathbf{0}}_{\mathrm{g}|\mathbf{c}}\big|\widehat{\mathbf{E}}_{\boldsymbol{\Psi},\xi}\big|\chi^{\mathrm{tot}\mathbf{0}}_{\mathrm{g}|\mathbf{c}}\rangle_{\mathbf{s}} + {}_{\mathbf{s}}\langle\chi^{\mathrm{tot}\mathbf{0}}_{\mathrm{g}|\mathbf{c}}\big|\widehat{\mathbf{E}}^{+}_{\boldsymbol{\Psi},\xi}\big|\chi^{\mathrm{tot}\mathbf{0}}_{\mathrm{g}|\mathbf{c}}\rangle_{\mathbf{s}} \right\}.$$

$$\widehat{E}_{\Psi,\xi} \stackrel{def}{=} \eta_0\Psi_H(\overline{\eta}_1^+,\pi_1,o_I;\beta,\xi) = \eta_0\overline{\eta}_1^+\left(l_1 + \mathcal{P}_1\eta_1\Big[(1+\beta)l_1^+ + \frac{2\beta}{2s-4+d}l_1l_{11}^+\Big] + \frac{\xi}{2}\pi_1\right),$$

for $(\varepsilon, gh_H, gh_L, gh_{\mathrm{tot}})\widehat{E}_{\Psi,\xi} = \vec{0}$, $[(\varepsilon, gh_H, gh_L, gh_{\mathrm{tot}})\Psi_H = (1,-1,0,-1)]$, containing for $\beta=0$ only the half from the Wick pairs of the operators o_I in a $\mathcal{A}_c(Y(1),\mathbf{R}^{1,d-1})$
For the Fronsdal formulation $(\Phi_{(\mu)_s})$ $\Psi\left(\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\right)$ passes to

$$\Psi\left(\Phi,\overline{C},b\right) = \left\{\langle\overline{C}(a)\big|\left(l_1\big|\Phi\rangle_s + \Big[(1+\beta)l_1^+ + \frac{2\beta}{2s-4+d}l_1l_{11}^+\Big]l_{11}\big|\Phi\rangle_s + \frac{\xi}{2}\big|b(a^+)\rangle_{s-1}\right) + h.c.\right\}$$

$$\sim (-1)^{s-1}2\int d^dx\overline{C}^{(\mu)_{s-1}}\left\{\partial^{\mu_s}\Phi_{(\mu)_s} - \Big[1+\beta+\frac{2\beta}{2s-4+d}\Big]\partial_{\mu_{s-1}}\Phi''_{(\mu)_{s-2}} + \frac{\xi}{2}b_{(\mu)_{s-1}}\right\}.$$

The Faddeev-Popov operator permits the representation

$$\widehat{\mathcal{M}}_{\Psi,c} \equiv \widehat{\mathcal{M}}_{\Psi,c}(\mathcal{C},\mathcal{P},\overline{\mathcal{C}},\overline{\mathcal{P}};a,a^+;x,y) \stackrel{def}{=} \big|\widehat{\chi}(\Phi(\eta_0),x,b,\beta,\xi)\rangle_s\frac{\overleftarrow{\delta}_{\eta_0}}{\delta\big|\chi^0_c(y)\rangle_s}Q_c \equiv \widehat{E}_{\Psi,\xi}Q_c$$

$$\Longrightarrow \int d^dy\widehat{\mathcal{M}}_{\Psi,c}(\ldots;x,y)\big|\chi^1_c(y)\rangle_s = \eta_0\overline{\eta}_1^+\eta_1\left(l_0 - \beta l_1^+l_1 - \frac{2\beta}{2s-4+d}l_1l_{11}^+l_1\right)\big|\chi^1_c(x)\rangle_s = 0,$$

Generating functionals of Green functions, BRST transforms, Ward identities

The Quantum Action

$$S_0^{\Psi}\left(\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\right) = S_0\left(S^{\Phi}, B^{\Phi}, S^{*} + \frac{\delta_{\eta_0}\Psi}{\delta S^{\Phi}}, B^{*}\right)$$

$$= \int d\eta_0\ {}_s\langle\chi^{\Psi\mathrm{tot}0}_{g|c}\big|Q_{c|\mathrm{tot}}\big|\chi^{\Psi\mathrm{tot}0}_{g|c}\rangle_s, \quad \text{for} \quad \big|\chi^{\Psi\mathrm{tot}0}_{g|c}\rangle_s = \big|S^{\mathrm{tot}0}_{\mathrm{g}|c}\rangle_s + \frac{\delta_{\eta_0}\Psi}{\delta_s\langle S^{\Phi}\big|} + \eta_0\big|B^{\mathrm{tot}0}_{\mathrm{g}|c}\rangle_s$$

should satisfy to the QME be non-degenerate and permits to determine Generating functional of Green's functions with new *generalized vector of the external sources* $\big|J^0_{\mathrm{g}|c}\rangle_s$ (and its dual ${}_s\langle J^0_{\mathrm{g}|c}|$) to the generalized field vector ${}_s\langle\chi^0_{\mathrm{g}|c}|$ (and its dual $\big|\chi^0_{\mathrm{g}|c}\rangle_s$) which contains usual sources $|J\rangle_s$, ${}_s\langle J|$ to field vectors ${}_s\langle\chi^0_c|$ (and its dual $\big|\chi^0_c\rangle_s$)

$$Z\left[J, \Phi^{*}\right] = \int d\big|\chi^0_{\mathrm{g}|c}\rangle_s d_s\langle\chi^0_{\mathrm{g}|c}|\exp\left\{\frac{\imath}{\hbar}\left(S_0^{\Psi}\left(\chi^{\mathrm{tot}0}_{\mathrm{g}|c}\right) + \int d\eta_0{}_s\langle J^0_{\mathrm{g}|c}|\chi^0_{\mathrm{g}|c}\rangle_s + h.c.\right)\right.$$

$$G^{(2)}(\ldots x, y) \sim \frac{\overrightarrow{\delta}}{\delta_s\langle J^0_{\mathrm{g}|c}|} Z\left[J, \Phi^{*}\right] \frac{\overleftarrow{\delta}}{\delta|J^0_{\mathrm{g}|c}\rangle_s}.$$

The respective Green's functions may be obtained by differentiation w.r.t. to external sources as wells as the Ward Identity for Z because of the BRST transformations $\delta_B\big|\chi^{\Psi 0}_{g|c}\rangle_s = \mu Q_{c|\mathrm{tot}}\big|C_c\rangle_s$ for integrand in Z for $J = 0$.

The work in progress and any collaborators are welcome!

Conclusions

- The constrained BRST-BFV method to construct gauge-invariant Lagrangian Formulations for mixed-symmetric half-integer HS fields subject to $Y(s_1, ..., s_k)$ in $\mathbb{R}^{1,d-1}$ space is suggested ;
- The equivalence among dynamics in constrained and unconstrained BRST-BFV Lagrangian Formulations for the (half)-integer MS HS field as well as with respective solutions for initial Poincare group irreps conditions are established;
- The constrained BRST-BV method to find minimal field-antifield BV actions with off-shell algebraic constraints for MS half-integer HS fields subject to $Y(s_1, ..., s_k)$ in $\mathbb{R}^{1,d-1}$ space is developed ;
- The Fang-Fronsdal, triplet formulations with off-shell (gamma-traceless) constraints for totally-symmetric of spin $(n + 1/2)$ HS field and unconstrained quartet GI Lagrangians are derived;
- The field-antifield minimal BV actions for BRST-BFV constrained Lagrangian formulations for half-integer MS HS fields subject to $Y(s_1, ..., s_k)$ are constructed;
- The gauge-fixing procedure to construct Quantum Action as extension of BRST-BV method by non-minimal constrained BRST-BFV .and BRST-BV variables are suggested for TS integer HS fields;
- the interaction problems within BRST-BV approach for TS integer HS fields and quantization rules are proposed.

Outlook

- development of the constrained BRST-BFV and BRST-BV Lagrangian formulations for HS fields on AdS_d starting from totally-symmetric cases to get Fang-Fronsdal, triplet, quartet formulations ;
- Construction on a base of constrained BRST-BFV(BV) Lagrangians for totally-symmetric HS fields with integer and half-integer spins the SUSY Lagrangian formulation (developed on component levels for some supermultiplets by S.Kusenko, J.Buchbinder, Yu.Zinoviev, T.Snegirev,....) for supermultiplet of given superspin(s) with triplets of integer and half-integer HS fields;
- Explicit Development of the interacting Lagrangians both for SUSY Lagrangian formulation (like Wess-Zumino model) and without SUSY but with constrained integer and half-integer HS fields.
- The collaborators to join with interaction and proposed quantization rules are very necessary, useful, interesting p

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Thank you very much