Asymptotic solutions to the nonlocal Fisher-KPP equation

Symmetries

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Outline

The Fisher–KPP equation

1. The aim: to consider symmetries and their solutions for a nonlocal generalization of the well-known Fisher–Kolmogorov–Petrovski–Piskunov (Fisher–KPP or FKPP) equation that belongs to the class of reaction-diffusion (RD) equations.

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1. The aim: to consider symmetries and their solutions for a nonlocal generalization of the well-known Fisher–Kolmogorov–Petrovski–Piskunov (Fisher–KPP or FKPP) equation that belongs to the class of reaction-diffusion (RD) equations.
2. The RD systems describe a vast class of physical phenomena ranging from high energy physics (HEP) and statistical physics to population dynamics in biophysics (populations of bacteria, viruses and cells).
3. In HEP a progress has been achieved in understanding high energy hard scattering at or near the unitarity limit on the base of equations proposed by Balitsky (Nucl. Phys. B 463 (1996) 99) and by Jalilian-Marian, Iancu, McLerran, Leonidov, Kovner and Weigert (JIMWLK)'1997-2001.

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2. The BK equation belongs to the same class as the Fisher–KPP equation.
3. It is used in study of the energy (rapidity Y) - saturation momentum dependences, $Q(Y)$, and geometric properties of the scattering characteristics (for details see, e.g, E. Iancu, A.H. Mueller, S. Munier Phys.Lett.B 606 (2005) 342).

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2. In statistical physics the branching Brownian motion, or BBM, introduced by Ikeda, Nagasawa, and Watanabe'1968, can be associated with the Fisher–KPP equation as shown by McKean (In: Commun. Pure App.Math. 28:3 (1975) 323).

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3. In mathematical biology the classical Fisher–KPP population equation (Fisher'1937; Kolmogorov, et al.'1937) describes space-time evolution of microbiological population density (bacteria or cells).

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3. In mathematical biology the classical Fisher–KPP population equation (Fisher'1937; Kolmogorov, et al.'1937) describes space-time evolution of microbiological population density (bacteria or cells).
4. The long-range interactions in the RD system require nonlocal generalizations of the FKPP equation (e.g., Fuentes, et al., 2003; Cunha et al., 2009).

IDEs and symmetries

1. The symmetry analysis of nonlinear integro-differential equations (IDEs) of reaction-diffusion type with nonlocal nonlinearity is is the actively developing area of symmetry analysis in mathematical physics.

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2. Advances of the group analysis were achieved in development of the theory of one- and multi-parameter approximate transformation groups initiated by Ibragimov, Baikov, Gazizov'1989 and other researchers.
3. Differ to these approaches, we study symmetries of nonlinear IDE's on the base of **combination of classical group analysis and Maslov's methods of semiclassical asymptotics.**

1. A special case of the Fisher–KPP equation with nonlocal nonlinearity:

$$[-\partial_t + \hat{H}_{nl}]u(x,t) = 0, \tag{2.1}$$

$$\hat{H}_{nl}[u](x,t) = Du_{xx}(x,t) + au(x,t) - m_u(t)\cdot u(x,t),$$

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2. In general case Eq. (2.1) has the nonlocal competition losses of the form $\int_{-\infty}^{\infty} b(x,y,t)u(y,t)dy$, $a > 0$ is a temp of autocatalytic reaction in the system, D is diffusion factor.

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3. Eq.(2.1) with $b(x,y,t) = k_0$, is a special case of the nonlocal Fisher–KPP equation obtained *in construction of semiclassical asymptotics* for the FKPP equation with competition losses $b(x,y,t)$ of general form.

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2. The moments are

$$x_u(t) = \frac{1}{m_u(t)} \int_{-\infty}^{\infty} x u(x,t) dx, \tag{3.3}$$

$$\alpha_u^{(k)}(t) = \frac{1}{m_u(t)} \langle \Delta x^k \rangle (t) =$$

$$= \frac{1}{m_u(t)} \int_{-\infty}^{\infty} (x - x_u(t))^k u(x,t) dx, \;\; k \geq 2. \tag{3.4}$$

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3. From Eq. (2.1) we obtain the EE system:

$$\dot{m}_u = \varphi(m_u) m_u, \ \ \varphi(m_u) = a - k_0 m_u, \tag{3.5}$$

$$\dot{x}_u = 0, \ \ \dot{\alpha}_u^{(k)} = Dk(k-1)\alpha_u^{(k-2)}. \tag{3.6}$$

Symmetries of the consistent system

- A symmetry of an equation $\hat{L}u = 0$ is defined as an operator $\hat{\sigma}$ such that
$$\hat{L}u = 0 \Rightarrow \hat{L}(u + \alpha\hat{\sigma}u) = o(\alpha), \tag{4.7}$$
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- **In the semiclassical approximation the Fisher–KPP equation with nonlocal nonlinearity is reduced to the consistent system(CS) formed by the reduced original Fisher–KPP equation, including nonlocal terms in the form of moments, and the EE dynamical system that is finite ODE system.**

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- **In the semiclassical approximation the Fisher–KPP equation with nonlocal nonlinearity is reduced to the consistent system(CS) formed by the reduced original Fisher–KPP equation, including nonlocal terms in the form of moments, and the EE dynamical system that is finite ODE system.**
- Example of the consistent system provide Eq. (2.1) and the ODE of moment system (2.2)–(3.4).

Symmetries and determining equations

- Symmetries are sought for as

$$\hat{\sigma}[u](x,t) = \sigma(x,t,u(x,t),u_x(x,t),\ldots,u_{x\ldots x},m_u(t)),$$
$$\vartheta(t,m_u(t)),\ \vartheta(t) = \int_{-\infty}^{\infty}\hat{\sigma}[u](x,t)dx,$$

$\sigma(x,t,u,u_1,\ldots,u_r,m_u)$ is the symbol of the operator $\hat{\sigma}$,
$x,t,u,u_1,u_2,\ldots,m_u$ are real variable.
Determining equations (in the operator form):

$$-\hat{\sigma}_t[u](x,t) + \hat{H}'_{nl}[u]\hat{\sigma}[u] - \hat{\sigma}'[u]\hat{H}_{nl}[u] = 0$$
$$-\vartheta_t(t,m_u(t)) + \varphi(m_u(t))\vartheta + \varphi'(m_u)\vartheta,$$

Lie symmetries

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$$\widehat{Y} = \sigma \frac{\partial}{\partial u} + \vartheta \frac{\partial}{\partial m_u}.$$

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- From $\sigma = \eta^1 - \xi^1 u_t - \xi^2 u_x$ and $\vartheta = \eta^2 - \dot{m}_u$, we obtain generator of the symmetry group of point transformations

$$\widehat{X} = \xi^1 \partial_t + \xi^2 \partial_x + \eta^1 \partial_u + \eta^2 \partial_{m_u}.$$

- Equations determining σ and ϑ:

$$\begin{aligned}
&\sigma_t + \varphi u \sigma_u + \varphi u_1 \sigma_{u_1} + \varphi u_2 \sigma_{u_2} - \varphi\sigma - D(\sigma_{xx} + 2u_1\sigma_{xu} + \\
&+2u_2\sigma_{xu_1} + 2u_3\sigma_{xu_2} + 2u_1u_2\sigma_{uu_1} + 2u_1u_3\sigma_{uu_2} + \\
&+2u_2u_3\sigma_{u_1u_2} + u_1^2\sigma_{uu} + u_2^2\sigma_{u_1u_1} + u_3^2\sigma_{u_2u_2}) + \\
&+\varphi m_u \sigma_{m_u} + k_0\vartheta u = 0, \qquad (5.8)
\end{aligned}$$

$$\vartheta_t + \varphi m_u \vartheta_{m_u} = -k_0\vartheta m_u + \varphi\vartheta. \qquad (5.9)$$

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$$\vartheta_t + \varphi m_u \vartheta_{m_u} = -k_0\vartheta m_u + \varphi\vartheta. \tag{5.9}$$

- With the change of variables

$$\tau = t - \frac{1}{a}\ln\frac{m_u}{a - k_0 m_u}, \quad z = \frac{1}{a}\ln\frac{m_u}{a - k_0 m_u}, \tag{5.10}$$

we have

$$\frac{\partial}{\partial t} + \varphi m_u \frac{\partial}{\partial m_u} = \frac{\partial}{\partial z}. \tag{5.11}$$

- Rewrite (5.8), (5.9) as

$$\begin{aligned}
&\sigma_z + \varphi u \sigma_u + \varphi u_1 \sigma_{u_1} + \varphi u_2 \sigma_{u_2} - \varphi \sigma - \\
&D(\sigma_{xx} + 2u_1\sigma_{xu} + 2u_2\sigma_{xu_1} + 2u_3\sigma_{xu_2} + \\
&+2u_1 u_2 \sigma_{uu_1} + 2u_1 u_3 \sigma_{uu_2} + 2u_2 u_3 \sigma_{u_1 u_2} + \\
&+u_1^2 \sigma_{uu} + u_2^2 \sigma_{u_1 u_1} + u_3^2 \sigma_{u_2 u_2}) + k_0 \vartheta u = 0, \quad (5.12)\\
&\vartheta_z = \vartheta \frac{a(1 - k_0 e^{az})}{1 + k_0 e^{az}}. \quad (5.13)
\end{aligned}$$

- Eq. (5.12) results in the overdetermined system of equations that is solved in the standard way.

General solution of the determining equations

- The general solution of Eq. (5.12) is

$$\sigma=\left(\frac{A_2}{a}\log\frac{m_u}{a-k_0m_u}+A_3\right)u_2+\left(\frac{1}{2D}A_2x+E_2\right)u_1+$$
$$+\left(\frac{1}{2D}A_2+S\frac{a-k_0m_u}{a^2}-\frac{m_R}{m_u}\right)u+R,$$
$$\vartheta=S\frac{m_u(a-k_0m_u)}{a^2},\quad m_R=\int_{-\infty}^{\infty}Rdx.$$

- Here, $A_2(t,m_u), A_3(t,m_u), E_2(t,m_u), S(t,m_u)$, are arbitrary functions of their arguments. Function $R=R(x,\tau,z)$ is a solution of the equation

$$R_z=DR_{xx}+\varphi R,$$

$$\tau=t-a^{-1}\log(m_u/(a-k_0m_u)),$$
$$z=a^{-1}\log(m_u/(a-k_0m_u)).$$

Basis symmetries

- The general solution (5.14) is a linear combination of the basis symmetries (σ, θ) given below by operator symbols:

$$
\begin{aligned}
&\sigma_1 = \frac{1}{a} \ln \frac{m_u}{\varphi} u_2 + \frac{1}{2D} x u_1 + \frac{1}{2D} u, \ \ \vartheta_1 = 0; \\
&\sigma_2 = \varphi u, \ \ \vartheta_2 = m_u \varphi; \\
&\sigma_3 = u_1, \ \ \vartheta_3 = 0; \\
&\sigma_4 = u_2, \ \ \vartheta_4 = 0; \\
&\sigma_5 = R - \frac{m_u}{m_R}, \ \ \vartheta_5 = 0.
\end{aligned}
$$

Generators of point symmetry group

- Generators of the symmetry Lie group of point transformations of the consistent system obtained from $\sigma_1, \ldots, \sigma_5$:

$$\widehat{X}_1 = \frac{1}{a}\log\frac{m_u}{\varphi}\partial_t + \frac{x}{2}\partial_x - \left(\frac{1}{2} - \frac{\varphi}{a}\log\frac{m_u}{\varphi}\right)u\partial_u + \\ + \frac{\varphi m_u}{a}\log\frac{m_u}{\varphi}\partial_{m_u},\ \ \widehat{X}_2 = \varphi u\partial_u + \varphi m_u\partial_{m_u}, \\ \widehat{X}_3 = \partial_x,\ \ \widehat{X}_4 = \partial_t,\ \ \widehat{X}_5 = \left(R - \frac{m_R}{m_u}u\right)\partial_u.$$

Invariance group of the consistent system

- Groups of transformations:

$$\widehat{X}_1 : x' = xe^{\frac{s}{2}}, \quad t' = t + \frac{1}{a}\log(m_u/\varphi)(e^s - 1),$$

$$u' = \frac{um'_u}{m_u}e^{-s/2}, m'_u = a\left(m_u/\varphi\right)^{e^s}\left(1 + k_0\left(m_u/\varphi\right)^{e^s}\right)^{-1},$$

$$\widehat{X}_2 : x' = x, \quad t' = t, \quad u' = \frac{aue^{sa}}{a - k_0 m_u + k_0 m_u e^{sa}},$$

$$m'_u = \frac{am_u e^{sa}}{a - k_0 m_u + k_0 m_u e^{sa}},$$

$$\widehat{X}_3 : x' = x + s, \quad t' = t, \quad u' = u, \quad m'_u = m_u,$$

$$\widehat{X}_4 : x' = x, \quad t' = t + s, \quad u' = u, \quad m'_u = m_u,$$

$$\widehat{X}_5 : x' = x, \quad t' = t, \quad u' = R\frac{m_u}{m_R}(1 - e^{-\frac{m_R}{m_u}s}) +$$

$$+ue^{-\frac{m_R}{m_u}s}, \ m'_u = m_u,$$

where s is a group parameter.

group-invariant solutions

- Group-invariant solutions are found from equations:

$$\left[-\partial_t + \hat{H}_{nl}[u] \right] u = 0, \tag{7.14}$$

$$\hat{\sigma}[u] = 0. \tag{7.15}$$

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- With the use of σ_1 we have

$$\alpha u_{xx} + x u_x + u = 0, \quad \alpha = \alpha(t) = \frac{2D}{a} \ln \frac{m_u(t)}{\varphi(t)}. \tag{7.16}$$

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- The two linear independent solutions are:

$$u_1(t,x) = \exp\left(-\frac{x^2}{2\alpha}\right),$$
$$u_2(t,x) = \exp\left(-\frac{x^2}{2\alpha}\right)\int_0^x \exp\left(\frac{\tilde{x}^2}{2\alpha}\right) d\tilde{x}.$$

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- With the use of Einstein–Ehrenfest equation (3.5) we obtain the group-invariant solution

$$u(x,t) = w_0\left(\frac{m_u\sqrt{\alpha_0}}{m_\gamma\sqrt{2Dt+\alpha_0}}\right)\exp\left(-\frac{x^2}{2(2Dt+\alpha_0)}\right), \tag{7.17}$$

where m_γ is an arbitrary constant, $m_\gamma \in \left(\frac{a}{k_0+1}, \frac{a}{k_0}\right)$,

$$\alpha_0 = \frac{2D}{a}\ln\frac{m_\gamma}{a-k_0 m_\gamma}, \tag{7.18}$$

$$w_0 = \frac{m_\gamma}{\sqrt{2\pi\alpha_0}}.$$

Summary

- In the framework of semiclassical approximation we reduce the original Fisher–KPP equation with nonlocal nonlinearity to a consistent system:
  (a) the reduced original nonlocal Fisher–KPP equation which *include nonlinearity in the form* of moments;
  (b) the finite approximation of dynamical system (of ODEs) governing evolution of moments of the equation solution.

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- Lie symmetries are calculated for the CS, group of invariance of the CS is obtained, and group-invariant solutions are found.

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- Lie symmetries are calculated for the CS, group of invariance of the CS is obtained, and group-invariant solutions are found.
- The group-invariant solution $u(x, t)$ of the form (7.17) belongs to the class of trajectory concentrated functions in which semiclassical asymptotics are constructed for the Fisher–KPP equation.