Deformed $\mathcal{N} = 8$ mechanics of $(\mathbf{8}, \mathbf{8}, \mathbf{0})$ multiplets

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The talk is based on:

Evgeny Ivanov, Olaf Lechtenfeld, Stepan Sidorov, `arXiv:1807.xxxxx [hep-th]`.

Outline

1. SU(4|1) supersymmetric mechanics
   - Relation to matrix models
2. The multiplet $(\mathbf{8}, \mathbf{8}, \mathbf{0})$
   - Chiral superspace description
   - SU(2|1) superfield approach
   - Superconformal action
3. The multiplet $(\mathbf{8}, \mathbf{8}, \mathbf{0})$: "mirror" counterpart
   - SU(2|1) superfield approach
4. Summary

SU(4|1) supersymmetric mechanics

We consider the superalgebra $su(4|1)$ as a deformation of the standard $\mathcal{N}=8$, $d=1$ superalgebra:

$$(\mathcal{N}=8,\ d=1) \quad \Rightarrow \quad su(4|1).$$

It is given by the following (anti)commutators:

$$\left\{Q^I, \bar{Q}_J\right\} = 2m\, L^I_J + 2\delta^I_J \mathcal{H}, \qquad \left[L^I_J, L^K_L\right] = \delta^K_J\, L^I_L - \delta^I_L L^K_J\,,$$
$$\left[L^I_J, Q^K\right] = \delta^K_J\, Q^I - \frac{1}{4}\,\delta^I_J Q^K, \qquad \left[L^I_J, \bar{Q}_L\right] = \frac{1}{4}\,\delta^I_J \bar{Q}_L - \delta^I_L \bar{Q}_J\,,$$
$$\left[\mathcal{H}, Q^K\right] = -\,\frac{3m}{4}\,Q^K, \qquad [\mathcal{H}, \bar{Q}_L] = \frac{3m}{4}\,\bar{Q}_L\,.$$

All other (anti)commutators are vanishing. Here, the superalgebra $su(4|1)$ contains eight supercharges and $\mathrm{SU}(4)\times\mathrm{U}(1)$ generators L^I_J, $\mathcal{H}$. It can be viewed as a deformation of the standard $\mathcal{N}=8$, $d=1$ superalgebra.

SU(4|1), $d = 1$ superspace

The SU(4|1), $d = 1$ superspace is defined as the coset superspace

$$\frac{\mathrm{SU}(4|1)}{\mathrm{SU}(4)} \sim \frac{\{Q^I, \bar{Q}_J, L^I_J, \mathcal{H}\}}{\{L^I_J\}}\,,$$

where its parameters define the superspace coordinates:

$$\zeta = \left\{t, \theta_I, \bar{\theta}^J\right\}, \qquad \overline{(\theta_I)} = \bar{\theta}^I.$$

Realization of the supergroup for the fermionic coset SU(n|1)/U(n) was studied in E. Ivanov, L. Mezincescu, A. Pashnev, P.K. Townsend, arXiv:hep-th/0301241. Extending this realization by time coordinate, we obtain the odd transformations:

$$\delta\theta_I = \epsilon_I + 2m\,\bar{\epsilon}^K\theta_K\theta_I\,, \qquad \delta\bar{\theta}^J = \bar{\epsilon}^J - 2m\,\epsilon_K\bar{\theta}^K\bar{\theta}^J, \qquad \delta t = i\left(\bar{\epsilon}^K\theta_K + \epsilon_K\bar{\theta}^K\right).$$

Deformed $\mathcal{N} = 8$ supermultiplets

- Employing the appropriate worldline superfield approach SU(2|2), we considered deformed analogs of $\mathcal{N} = 8$ supersymmetric quantum mechanics (Evgeny Ivanov, Olaf Lechtenfeld, Stepan Sidorov, arXiv:1609.00490 [hep-th]). We studied models of SU(2|2) supersymmetric mechanics based on the off-shell multiplets $(\mathbf{3}, \mathbf{8}, \mathbf{5})$, $(\mathbf{4}, \mathbf{8}, \mathbf{4})$ and $(\mathbf{5}, \mathbf{8}, \mathbf{3})$.

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- Other multiplets $(\mathbf{k}, \mathbf{8}, \mathbf{8} - \mathbf{k})$, $(\mathbf{k} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{6}, \mathbf{7}, \mathbf{8})$ of the standard $\mathcal{N} = 8$ mechanics have no deformations to SU(2|2) mechanics. They have generalizations to SU(4|1) supersymmetric mechanics.

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- The multiplet $(\mathbf{4},\mathbf{8},\mathbf{4})$ is exceptional: none of $\mathcal{N}=8$, $d=1$ superconformal symmetries can act on it. However, one can realize on it an $\mathcal{N}=8$ extended Heisenberg superalgebra given in S. Bellucci, E. Ivanov, A. Sutulin, arXiv:hep-th/0504185. This extended superalgebra contains an $su(2|2)$ superalgebra.

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- The superalgebra $su(2|2)$ can be embedded only into the superconformal algebra $osp(4^*|4)$. Hence, the supergroup $\mathrm{SU}(2|2)$ admits an action only on the multiplets $(\mathbf{3},\mathbf{8},\mathbf{5})$ and $(\mathbf{5},\mathbf{8},\mathbf{3})$.

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- The superalgebra $su(2|2)$ can be embedded only into the superconformal algebra $osp(4^*|4)$. Hence, the supergroup $\mathrm{SU}(2|2)$ admits an action only on the multiplets $(\mathbf{3}, \mathbf{8}, \mathbf{5})$ and $(\mathbf{5}, \mathbf{8}, \mathbf{3})$.
- The superalgebra $su(4|1)$ can be embedded into the rest of superconformal algebras $osp(8|2)$, $F(4)$, $su(4|1, 1)$. Thus, the multiplets $(\mathbf{0}, \mathbf{8}, \mathbf{8})$, $(\mathbf{1}, \mathbf{8}, \mathbf{7})$, $(\mathbf{2}, \mathbf{8}, \mathbf{6})$, $(\mathbf{6}, \mathbf{8}, \mathbf{2})$, $(\mathbf{7}, \mathbf{8}, \mathbf{1})$ and $(\mathbf{8}, \mathbf{8}, \mathbf{0})$ have generalizations to $\mathrm{SU}(4|1)$ supersymmetric mechanics.

Relation to matrix models

- Berenstein, Maldacena and Nastase (BMN) proposed a matrix model associated with M-theory on pp-wave background D. Berenstein, J. Maldacena, H. Nastase, arXiv:hep-th/0202021 with 16 supersymmetries corresponding to the mass-deformed world-line supersymmetry group SU(4|2). Their (on-shell) multiplet is given by

$$\boxed{\left(y^{IJ}, v^{\alpha\beta}, \chi^{I\alpha}, \bar{\chi}_{I\alpha}\right), \qquad y^{IJ} \equiv y^{[IJ]}, \qquad v^{\alpha\beta} \equiv v^{(\alpha\beta)},}$$

$$\overline{(y^{IJ})} = y_{IJ} = \frac{1}{2}\,\varepsilon_{IJKL}\,y^{KL}, \qquad \overline{(v^{\alpha\beta})} = v_{\alpha\beta}, \qquad \overline{(\chi^{I\alpha})} = \bar{\chi}_{I\alpha}\,,$$

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- It spurred investigations of massive matrix models of SQM with 8 supersymmetries corresponding to the groups SU(2|2), SU(4|1) and with 4 supersymmetries corresponding to the group SU(2|1).

Relation to matrix models

- Our aim here is to consider $(\mathbf{8},\mathbf{8},\mathbf{0})$ multiplets of the deformed supersymmetric mechanics with respect to the appropriate worldline realization of the supergroup SU(4|1):

$$\boxed{\left(\phi\,,\bar{\phi}\,,y^{IJ},\chi^{I},\bar{\chi}_{I}\right), \qquad \left(z^{I},\bar{z}_{I}\,,\chi\,,\bar{\chi}\,,\chi^{IJ}\right).}$$

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- SU(4|1) supersymmetric model corresponding to the multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$ were obtained from BMN matrix model in L. Motl, A. Neitzke, M.M. Sheikh-Jabbari, arXiv:hep-th/0306051.

The multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$

The first version of the multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$ is defined by the $\mathrm{SU}(4|1)$ covariant constraints

$$\bar{\mathcal{D}}_J\,\Phi = 0\,, \qquad \mathcal{D}^I\,\bar{\Phi} = 0\,, \qquad \bar{\mathcal{D}}_I\bar{\mathcal{D}}_J\,\bar{\Phi} = \frac{1}{2}\,\varepsilon_{IJKL}\,\mathcal{D}^K\mathcal{D}^L\,\Phi\,,$$
$$\sqrt{2}\,\mathcal{D}^I\,Y^{JK} = -\,\varepsilon^{IJKL}\,\bar{\mathcal{D}}_L\,\bar{\Phi}\,, \qquad \sqrt{2}\,\bar{\mathcal{D}}_J\,Y_{KL} = \varepsilon_{IJKL}\,\mathcal{D}^I\,\Phi\,,$$
$$\overline{(Y^{IJ})} = Y_{IJ} = \frac{1}{2}\,\varepsilon_{IJKL}\,Y^{KL}, \qquad \overline{(\Phi)} = \bar{\Phi}\,,$$

where Φ is a chiral superfield and Y^{IJ} is a superfield with antisymmetric $\mathrm{SU}(4)$ indices. Note that in the flat limit, when $m \to 0$ and

$$D^I = \frac{\partial}{\partial\theta_I} - i\bar{\theta}^I\partial_t\,, \qquad \bar{D}_J = -\,\frac{\partial}{\partial\bar{\theta}^J} + i\theta_J\partial_t\,,$$

this set of constraints becomes the set of superfield constraints defining the standard $\mathcal{N}=8$, $d=1$ multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$, such that only $\mathrm{SU}(4)\subset\mathrm{SO}(8)$ is manifest.

Chiral superspace description

The supergroup SU(4|1) admits two mutually conjugated complex supercosets which can be identified with the left and right chiral subspaces:

$$\zeta_{\rm L} = (t_{\rm L}, \theta_I)\,, \qquad \zeta_{\rm R} = \left(t_{\rm R}, \bar\theta^J\right).$$

The left even coordinate $t_{\rm L}$ is related to the real time coordinate t via

$$t_{\rm L} = t + \frac{i}{2m}\,\log\left(1 + 2m\,\bar\theta^K\theta_K\right).$$

Then we obtain the left chiral space $\zeta_{\rm L}$ closed under the supersymmetry transformations

$$\delta\theta_I = \epsilon_I + 2m\,\bar\epsilon^K\theta_K\theta_I\,, \qquad \delta t_{\rm L} = 2i\bar\epsilon^K\theta_K\,.$$

The left chiral measure is defined as

$$d\zeta_{\rm L} := dt_{\rm L}\,d^4\theta\,e^{-3imt_{\rm L}}, \qquad \delta\left(d\zeta_{\rm L}\right) = 0\,,$$
$$\int d\zeta_{\rm L}\,\theta_I\theta_J\theta_K\theta_L\,e^{3imt_{\rm L}} = \varepsilon_{IJKL}\,.$$

Chiral superfield

We consider the chiral superfield Φ given by the general θ-expansion

$$\begin{aligned}\Phi\left(t_{\mathrm{L}},\theta_I\right) &= \phi+\sqrt{2}\,\theta_K\chi^K e^{3imt_{\mathrm{L}}/4}+\theta_I\theta_J A^{IJ}e^{3imt_{\mathrm{L}}/2}+\frac{\sqrt{2}}{3}\,\theta_I\theta_J\theta_K\xi^{IJK}e^{9imt_{\mathrm{L}}/4}\\ &+\frac{1}{4}\,\varepsilon^{IJKL}\,\theta_I\theta_J\theta_K\theta_L B\,e^{3imt_{\mathrm{L}}},\quad A^{IJ}\equiv A^{[IJ]},\quad \xi^{IJK}\equiv\xi^{[IJK]}.\end{aligned}$$

The superfield Φ transforms as a singlet of the stability subgroup $\mathrm{SU}(4)$, *i.e.* $\delta\Phi=0$. Its components transformations:

$$\begin{aligned}
&\delta\phi=-\sqrt{2}\,\epsilon_K\chi^K e^{3imt/4},\\
&\delta\chi^I=\sqrt{2}\,\bar{\epsilon}^I\left(i\dot{\phi}\right)e^{-3imt/4}-\sqrt{2}\,\epsilon_K A^{IK}e^{3imt/4},\\
&\delta A^{IJ}=2\sqrt{2}\,\bar{\epsilon}^{[I}\left(i\dot{\chi}^{J]}+\frac{m}{4}\,\chi^{J]}\right)e^{-3imt/4}-\sqrt{2}\,\epsilon_K\xi^{IJK}e^{3imt/4},\\
&\frac{\sqrt{2}}{3}\,\delta\xi^{IJK}=2\,\bar{\epsilon}^{[K}\left(i\dot{A}^{IJ]}+\frac{m}{2}\,A^{IJ]}\right)e^{-3imt/4}-\varepsilon^{IJKL}\,\epsilon_L B\,e^{3imt/4},\\
&\varepsilon^{IJKL}\,\delta B=\frac{8\sqrt{2}}{3}\,\bar{\epsilon}^{[L}\left(i\dot{\xi}^{IJK]}+\frac{3m}{4}\,\xi^{IJK]}\right)e^{-3imt/4}.
\end{aligned}$$

Additional constraints

As the next step, we give the rest of constraints in the component level requiring the field content to be $(\mathbf{8},\mathbf{8},\mathbf{0})$. Components of the chiral superfield Φ are subjected to the additional constraints

$$
\begin{aligned}
& A^{IJ} = \sqrt{2}\left(i\dot{y}^{IJ} - \frac{m}{2}\,y^{IJ}\right), \qquad \overline{(y^{IJ})} = y_{IJ} = \frac{1}{2}\,\varepsilon_{IJKL}\,y^{KL}, \\
& \xi^{IJK} = -\,\varepsilon^{IJKL}\left(i\dot{\bar{\chi}}_L - \frac{5m}{4}\,\bar{\chi}_L\right), \qquad \overline{(\chi^I)} = \bar{\chi}_I\,, \\
& B = \frac{2}{3}\left(\ddot{\bar{\phi}} + 2im\dot{\bar{\phi}}\right).
\end{aligned}
$$

It gives the following transformations:

$$
\begin{aligned}
& \delta\phi = -\sqrt{2}\,\epsilon_I\chi^I e^{3imt/4}, \qquad \delta\bar{\phi} = \sqrt{2}\,\bar{\epsilon}^I\,\bar{\chi}_I\,e^{-3imt/4}, \\
& \delta y^{IJ} = -\,2\,\bar{\epsilon}^{[I}\chi^{J]}e^{-3imt/4} + \varepsilon^{IJKL}\epsilon_K\bar{\chi}_L\,e^{3imt/4}, \\
& \delta\chi^I = \sqrt{2}\,\bar{\epsilon}^I\left(i\dot{\phi}\right)e^{-3imt/4} - 2\,\epsilon_J\left(i\dot{y}^{IJ} - \frac{m}{2}\,y^{IJ}\right)e^{3imt/4}, \\
& \delta\bar{\chi}_I = -\sqrt{2}\,\epsilon_I\left(i\dot{\bar{\phi}}\right)e^{3imt/4} + 2\,\bar{\epsilon}^J\left(i\dot{y}_{IJ} + \frac{m}{2}\,y_{IJ}\right)e^{-3imt/4}.
\end{aligned}
$$

The SU(4|1) invariant superfield action of the multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$ is written as

$$S_{\rm SK} = \int dt\,\mathcal{L}_{\rm SK} = -\,\frac{1}{4}\left[\int d\zeta_{\rm L}\,K\left(\Phi\right) + \int d\zeta_{\rm R}\,\bar{K}\left(\bar{\Phi}\right)\right].$$

Its component Lagrangian reads

$$\begin{aligned}
\mathcal{L}_{\rm SK} \;\;=\;\; & g_1\left[\dot{\phi}\dot{\bar{\phi}} + \frac{1}{2}\,\dot{y}^{IJ}\dot{y}_{IJ} + \frac{i}{2}\left(\chi^{K}\dot{\bar{\chi}}_{K} - \dot{\chi}^{K}\bar{\chi}_{K}\right) - \frac{5m}{4}\,\chi^{K}\bar{\chi}_{K} - \frac{m^2}{8}\,y^{IJ}y_{IJ}\right] \\
& - \frac{im}{4}\left(\dot{\phi}\,\partial_{\phi}g_1 - \dot{\bar{\phi}}\,\partial_{\bar{\phi}}g_1\right) y^{IJ}y_{IJ} + 2im\left(\dot{\phi}\,\partial_{\bar{\phi}}\bar{K} - \dot{\bar{\phi}}\,\partial_{\phi}K\right) \\
& + \frac{1}{\sqrt{2}}\left(i\dot{y}_{IJ} - \frac{m}{2}\,y_{IJ}\right)\chi^{I}\chi^{J}\,\partial_{\phi}g_1 + \frac{1}{\sqrt{2}}\left(i\dot{y}^{IJ} + \frac{m}{2}\,y^{IJ}\right)\bar{\chi}_{I}\bar{\chi}_{J}\,\partial_{\bar{\phi}}g_1 \\
& - \frac{i}{2}\left(\dot{\phi}\,\partial_{\phi}g_1 - \dot{\bar{\phi}}\,\partial_{\bar{\phi}}g_1\right)\chi^{K}\bar{\chi}_{K} - \frac{1}{24}\,\varepsilon^{IJKL}\,\bar{\chi}_{I}\bar{\chi}_{J}\bar{\chi}_{K}\bar{\chi}_{L}\,\partial_{\bar{\phi}}\partial_{\bar{\phi}}g_1 \\
& - \frac{1}{24}\,\varepsilon_{IJKL}\,\chi^{I}\chi^{J}\chi^{K}\chi^{L}\,\partial_{\phi}\partial_{\phi}g_1\,.
\end{aligned}$$

The complex fields ϕ is corresponding coordinate of Special Kähler manifold with the metric

$$g_1\left(\phi,\bar{\phi}\right) = \partial_{\phi}\partial_{\phi}K\left(\phi\right) + \partial_{\bar{\phi}}\partial_{\bar{\phi}}\bar{K}\left(\bar{\phi}\right).$$

Harmonic superspace description

- The multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$ has a description also in terms of the harmonic superfield $Y^{(+2)}$ defined on $\mathrm{SU}(4)/[\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)]$ type harmonic space (E. Ivanov, S. Kalitzin, A.V. Nguyen, V. Ogievetsky, J. Phys. A **18** (1985) 3433).

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- The analytic superspace

$$\zeta_{\rm A} = \left\{ t_{\rm A}, \theta^{(+)}_{a}, \bar\theta^{(+)i}, u^{(+)I}_{a}, u^{(+)i}_{I}, u^{(-)a}_{I}, u^{(-)I}_{i} \right\}.$$

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- The unitarity and unimodularity conditions are written as

$$u^{(+)i}_{K} u^{(-)K}_{j} = \delta^{i}_{j}\,, \qquad u^{(-)a}_{K} u^{(+)K}_{b} = \delta^{a}_{b}\,, \qquad u^{(-)a}_{J} u^{(+)I}_{a} + u^{(+)i}_{J} u^{(-)I}_{i} = \delta^{I}_{J}\,,$$
$$u^{(-)a}_{K} u^{(-)K}_{j} = u^{(+)i}_{K} u^{(+)K}_{b} = 0\,, \quad \varepsilon^{IJKL}\varepsilon_{ij}\, u^{(+)i}_{K} u^{(+)j}_{L} + 2\,\varepsilon^{ab} u^{(+)I}_{a} u^{(+)J}_{b} = 0\,.$$

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- The analytic superspace

$$\zeta_{\mathrm{A}} = \Big\{ t_{\mathrm{A}}, \theta_a^{(+)}, \bar\theta^{(+)i}, u_a^{(+)I}, u_I^{(+)i}, u_I^{(-)a}, u_i^{(-)I} \Big\}.$$

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$$u_K^{(+)i} u_j^{(-)K} = \delta_j^i\,, \qquad u_K^{(-)a} u_b^{(+)K} = \delta_b^a\,, \qquad u_J^{(-)a} u_a^{(+)I} + u_J^{(+)i} u_i^{(-)I} = \delta_J^I\,,$$
$$u_K^{(-)a} u_j^{(-)K} = u_K^{(+)i} u_b^{(+)K} = 0\,, \quad \varepsilon^{IJKL}\varepsilon_{ij}\, u_K^{(+)i} u_L^{(+)j} + 2\,\varepsilon^{ab} u_a^{(+)I} u_b^{(+)J} = 0\,.$$

- The relevant analytic harmonic superfield is defined as

$$\mathcal{D}_a^{(+2)i} Y^{(+2)} = 0\,, \qquad \mathcal{D}_j^i Y^{(+2)} = \mathcal{D}_b^a Y^{(+2)} = 0\,.$$

The rest of constraints can be given by requiring the field content to be $(\mathbf{8},\mathbf{8},\mathbf{0})$ at the component level.

SU(2|1) superfield approach

- For more general construction of SU(4|1) invariant actions, it is convenient to employ SU(2|1) superfield approach. So, we split the multiplet $(\mathbf{8}, \mathbf{8}, \mathbf{0})$ into SU(2|1) multiplets as a sum of the ordinary multiplet $(\mathbf{4}, \mathbf{4}, \mathbf{0})$ and the “mirror” multiplet $(\mathbf{4}, \mathbf{4}, \mathbf{0})$ (E. Ivanov, S. Sidorov, arXiv:1507.00987 [hep-th]).

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- One can consider reducing of the SU(4|1) superspace to the SU(2|1) one. The SU(2|1) superspace coordinates as

$$\left\{t, \theta_i, \bar{\theta}^i\right\}, \qquad \overline{(\theta_i)} = \bar{\theta}^i, \quad i = 1, 2.$$

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$$\left\{t, \theta_i, \bar{\theta}^i\right\}, \qquad \overline{(\theta_i)} = \bar{\theta}^i, \quad i = 1, 2.$$

- Choosing ϵ_1 and ϵ_2 transformations, we obtain the SU(2|1) supersymmetric transformations:

$$\delta\theta_i = \epsilon_i + 2m\,\bar{\epsilon}^k\theta_k\theta_i\,, \quad \delta\bar{\theta}^j = \bar{\epsilon}^j - 2m\,\epsilon_k\bar{\theta}^k\bar{\theta}^j, \quad \delta t = i\left(\bar{\epsilon}^k\theta_k + \epsilon_k\bar{\theta}^k\right).$$

Subalgebra

The superalgebra $su(4|1)$ contains as subalgebra the extended $su(2|1) \niplus u(1)$ superalgebra:

$$\left\{Q^i, \bar{Q}_j\right\} = 2m\, I^i_j + m\,\delta^i_j F + 2\delta^i_j \mathcal{H}, \qquad \left[I^i_j, I^k_l\right] = \delta^k_j I^i_l - \delta^i_l I^k_j,$$
$$\left[I^i_j, Q^k\right] = \delta^k_k Q^i - \frac{1}{2}\,\delta^i_j Q^k, \qquad \left[I^i_j, \bar{Q}_l\right] = \frac{1}{2}\,\delta^i_j \bar{Q}_l - \delta^i_l \bar{Q}_j\,,$$
$$\left[\mathcal{H}, Q^k\right] = -\,\frac{3m}{4}\,Q^k, \qquad \left[\mathcal{H}, \bar{Q}_l\right] = \frac{3m}{4}\,\bar{Q}_l\,,$$
$$\left[F, Q^k\right] = \frac{1}{2}\,Q^k, \qquad \left[F, \bar{Q}_l\right] = -\,\frac{1}{2}\,\bar{Q}_l\,.$$

Here, SU(2) generators of $su(2|1)$ are defined as

$$I^i_j = L^i_j - \frac{1}{2}\,\delta^i_j F,$$

an internal U(1) generator of $su(2|1)$ by the combination

$$\tilde{H} = \mathcal{H} + \frac{m}{2}\,F,$$

where F is an external U(1) generator.

Splitting $(\mathbf{4}, \mathbf{4}, \mathbf{0}) \oplus (\mathbf{4}, \mathbf{4}, \mathbf{0})$

- The ordinary multiplet $(\mathbf{4}, \mathbf{4}, \mathbf{0})$ is described the superfield q^{ia} that obeys the SU(2|1) covariant constraints

$$\mathcal{D}^{(k} q^{i)a} = \bar{\mathcal{D}}^{(k} q^{i)a} = 0\,, \qquad \tilde{F} q^{ia} = 0\,, \qquad \overline{(q^{ia})} = q_{ia}\,.$$

Here, $\mathcal{D}^k$ and $\bar{\mathcal{D}}^k$ are SU(2|1) covariant derivatives. The indices $i = 1, 2$ and $a = 1, 2$ correspond to the fundamental indices of the subgroup $\mathrm{SU}(2) \times \mathrm{SU}(2) \subset \mathrm{SU}(4)$, respectively.

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- Corresponding SU(2|1) constraints defining the "mirror" $(\mathbf{4},\mathbf{4},\mathbf{0})$ multiplet are written as

$$\bar{\mathcal{D}}^i Z = \bar{\mathcal{D}}^i Y = 0\,, \qquad \mathcal{D}^i \bar{Z} = \mathcal{D}^i \bar{Y} = 0\,,$$
$$\mathcal{D}^i Z = -\,\bar{\mathcal{D}}^i \bar{Y}, \quad \mathcal{D}^i Y = \bar{\mathcal{D}}^i \bar{Z}, \qquad \tilde{F}Z = 0\,, \quad \tilde{F}Y = Y.$$

SU(2|1) superfield approach

Alternatively, we can employ the construction of SU(4|1) invariant actions in the framework of the SU(2|1) superfields q^{ia}, Y, Z. The general SU(2|1) superfield action is given by

$$S = \int dt\, d^2\theta\, d^2\bar{\theta} \left(1 + 2m\,\bar{\theta}^k\theta_k\right) f\left(Z, \bar{Z}, Y\bar{Y}, q^{ia}q_{ia}\right).$$

The metric g of target space is defined according to E. Ivanov, O. Lechtenfeld, A. Sutulin, arXiv:0705.3064 [hep-th] as

$$g = \Delta_2 f = -\,\Delta_1 f\,, \qquad f = f\left(z, \bar{z}, y\bar{y}, x^{ia}x_{ia}\right), \qquad g = g\left(z, \bar{z}, y\bar{y}, x^{ia}x_{ia}\right),$$
$$\Delta_1 f + \Delta_2 f = 0 \quad \Rightarrow \quad \Delta_1 g + \Delta_2 g = 0\,,$$
$$\Delta_1 = \varepsilon^{ik}\varepsilon^{ab}\partial_{ia}\partial_{kb}\,, \qquad \Delta_2 = 2\left(\partial_z\partial_{\bar{z}} + \partial_y\partial_{\bar{y}}\right).$$

Since SU(4) and SU(2|1) transformations are closed on SU(4|1) transformations, we require SU(4) invariance of the corresponding component action. Then we obtain the equation

$$\boxed{m\left(\bar{y}g + 2\partial_y f + x^{ia}\partial_{ia}\partial_y f\right) = 0 \quad \Rightarrow \quad m\left(x_{ia}\partial_y - \bar{y}\partial_{ia}\right) g = 0\,, \qquad \text{c.c.}}$$

Solutions

The equation gives three solutions:

1) Special Kähler manifold metric (Chiral superfield solution)

$$f_1 = \frac{1}{2}\left[\bar{z}\,\partial_z K\left(z\right) + z\,\partial_{\bar{z}}\bar{K}\left(\bar{z}\right)\right] - \left(\frac{x^{ia}x_{ia}}{16} + \frac{y\bar{y}}{4}\right)\left[\partial_z\partial_z K\left(z\right) + \partial_{\bar{z}}\partial_{\bar{z}}\bar{K}\left(\bar{z}\right)\right],$$

$$g_1 = \frac{1}{2}\left[\partial_z\partial_z K\left(z\right) + \partial_{\bar{z}}\partial_{\bar{z}}\bar{K}\left(\bar{z}\right)\right] \quad\Longrightarrow\quad g_1 = \partial_\phi\partial_\phi K\left(\phi\right) + \partial_{\bar{\phi}}\partial_{\bar{\phi}}\bar{K}\left(\bar{\phi}\right).$$

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2) SO(6)-invariant metric (Harmonic superfield solution)

$$f_2 = \frac{1}{4}\left(x^{ia}x_{ia}\right)^{-1}\log\left(2y\bar{y} + x^{ia}x_{ia}\right),$$

$$g_2 = \left(2y\bar{y} + x^{ia}x_{ia}\right)^{-2} \quad \Longrightarrow \quad g_2 = \left[\frac{1}{2}\,y^{IJ}y_{IJ}\right]^{-2}.$$

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3) SO(8)-invariant metric (OSp(8|2) superconformal solution)

$$f_3 = -\frac{1}{8}\left(x^{ia}x_{ia}\right)^{-1}\left(2z\bar z + 2y\bar y + x^{ia}x_{ia}\right)^{-1},$$

$$g_3 = \left(2z\bar z + 2y\bar y + x^{ia}x_{ia}\right)^{-3} \quad \Longrightarrow \quad g_3 = \left[\phi\bar\phi + \frac{1}{2}\,y^{IJ}y_{IJ}\right]^{-3}.$$

OSp(8|2) superconformal Lagrangian

OSp(8|2) superconformal Lagrangian of the trigonometric type contains only m^2 deformed terms:

$$
\begin{aligned}
\mathcal{L}_{\rm conf} \;=\; & g_3\left[\dot{\phi}\dot{\bar{\phi}} + \frac{1}{2}\,\dot{y}^{IJ}\dot{y}_{IJ} + \frac{i}{2}\left(\chi^K\dot{\bar{\chi}}_K - \dot{\chi}^K\bar{\chi}_K\right) - \frac{m^2}{4}\left(\phi\bar{\phi} + \frac{1}{2}\,y^{IJ}y_{IJ}\right)\right] \\
& - \frac{i}{\sqrt{2}}\,\dot{\bar{\phi}}\,\partial_{IJ}g_3\,\chi^I\chi^J - \frac{i}{\sqrt{2}}\,\dot{\phi}\,\partial^{IJ}g_3\,\bar{\chi}_I\bar{\chi}_J + i\left(\dot{y}_{IK}\,\partial^{JK}g_3 - \dot{y}^{JK}\,\partial_{IK}g\right)\chi^I\bar{\chi}_J \\
& + \frac{i}{\sqrt{2}}\left(\dot{y}_{IJ}\,\chi^I\chi^J\,\partial_\phi g_3 + \dot{y}^{IJ}\,\bar{\chi}_I\bar{\chi}_J\,\partial_{\bar{\phi}}g_3\right) - \frac{i}{2}\left(\dot{\phi}\,\partial_\phi g_3 - \dot{\bar{\phi}}\,\partial_{\bar{\phi}}g_3\right)\chi^K\bar{\chi}_K \\
& - \frac{1}{24}\left(\varepsilon_{IJKL}\,\chi^I\chi^J\chi^K\chi^L\,\partial_\phi\partial_\phi g_3 + \varepsilon^{IJKL}\,\bar{\chi}_I\bar{\chi}_J\bar{\chi}_K\bar{\chi}_L\,\partial_{\bar{\phi}}\partial_{\bar{\phi}}g_3\right) \\
& - \frac{1}{\sqrt{2}}\left(\chi^I\chi^J\,\partial_{IJ}\partial_\phi g_3 + \bar{\chi}_I\bar{\chi}_J\,\partial^{IJ}\partial_{\bar{\phi}}g_3\right)\chi^K\bar{\chi}_K \\
& - \frac{1}{2}\,\partial_{IJ}\partial^{KL}g_3\,\chi^I\chi^J\bar{\chi}_K\bar{\chi}_L + \frac{1}{2}\,\partial_\phi\partial_{\bar{\phi}}g_3\,\chi^I\bar{\chi}_I\chi^J\bar{\chi}_J\,.
\end{aligned}
$$

We have eliminated all deformed terms proportional to m in Lagrangian of the third solution by redefining the component fields as

$$
\begin{aligned}
\phi &\to \phi\, e^{-imt/2}, \qquad & \chi^I &\to \chi^I\, e^{-imt/4}, \\
\bar{\phi} &\to \bar{\phi}\, e^{imt/2}, \qquad & \bar{\chi}_I &\to \bar{\chi}_I\, e^{imt/4}.
\end{aligned}
$$

Superconformal transformations

Since this Lagrangian is an even function of m, it is invariant under two types of SU(4|1) transformations with the deformation parameters m and $-m$:

m

$$\begin{aligned}
&\delta\phi = -\sqrt{2}\,\epsilon_I\chi^I e^{imt}, \qquad \delta\bar{\phi} = \sqrt{2}\,\bar{\epsilon}^I\bar{\chi}_I\, e^{-imt},\\
&\delta y^{IJ} = -2\,\bar{\epsilon}^{[I}\chi^{J]}e^{-imt} + \varepsilon^{IJKL}\epsilon_K\bar{\chi}_L\, e^{imt},\\
&\delta\chi^I = \sqrt{2}\,\bar{\epsilon}^I\left(i\dot{\phi} + \frac{m}{2}\,\phi\right)e^{-imt} - 2\,\epsilon_J\left(i\dot{y}^{IJ} - \frac{m}{2}\,y^{IJ}\right)e^{imt},\\
&\delta\bar{\chi}_I = -\sqrt{2}\,\epsilon_I\left(i\dot{\bar{\phi}} - \frac{m}{2}\,\bar{\phi}\right)e^{imt} + 2\,\bar{\epsilon}^J\left(i\dot{y}_{IJ} + \frac{m}{2}\,y_{IJ}\right)e^{-imt},
\end{aligned}$$

$-m$

$$\begin{aligned}
&\delta\phi = -\sqrt{2}\,\eta_I\chi^I e^{-imt}, \qquad \delta\bar{\phi} = \sqrt{2}\,\bar{\eta}^I\bar{\chi}_I\, e^{imt},\\
&\delta y^{IJ} = -2\,\bar{\eta}^{[I}\chi^{J]}e^{imt} + \varepsilon^{IJKL}\eta_K\bar{\chi}_L\, e^{-imt},\\
&\delta\chi^I = \sqrt{2}\,\bar{\eta}^I\left(i\dot{\phi} - \frac{m}{2}\,\phi\right)e^{imt} - 2\,\eta_J\left(i\dot{y}^{IJ} + \frac{m}{2}\,y^{IJ}\right)e^{-imt},\\
&\delta\bar{\chi}_I = -\sqrt{2}\,\eta_I\left(i\dot{\bar{\phi}} + \frac{m}{2}\,\bar{\phi}\right)e^{-imt} + 2\,\bar{\eta}^J\left(i\dot{y}_{IJ} - \frac{m}{2}\,y_{IJ}\right)e^{imt}.
\end{aligned}$$

In the closure of these transformations, we obtain superconformal algebra $osp(8|2)$ composed of 16 supercharges and 31 bosonic generators. This property with respect to the deformed $su(2|1)$ and superconformal $d(2,1;\alpha)$ algebras was marked in E. Ivanov, S. Sidorov, F. Toppan, arXiv:1501.05622 [hep-th].

The multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$: “mirror” counterpart

- The mirror version of the multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$ is described by a complex bosonic superfield V^I satisfying

$$\mathcal{D}^I V^J = \frac{1}{2}\,\varepsilon^{IJKL}\,\bar{\mathcal{D}}_K \bar{V}_L\,, \qquad \mathcal{D}^{(I}\,V^{J)} = 0\,, \qquad \bar{\mathcal{D}}_{(K}\,\bar{V}_{L)} = 0\,,$$
$$\mathcal{D}^I \bar{V}_J = \frac{1}{4}\,\delta^I_J \mathcal{D}^K \bar{V}_K \qquad \bar{\mathcal{D}}_J\,V^I = \frac{1}{4}\,\delta^I_J \bar{\mathcal{D}}_K V^K \qquad \overline{(V^I)} = \bar{V}_I\,.$$

In the flat limit $m \to 0$, these constraints correspond to the $\mathrm{SU}(4)$ covariant constraints defining another form of the standard $\mathcal{N}=8$, $d=1$ multiplet $(\mathbf{8},\mathbf{8},\mathbf{0})$.

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- Avoiding calculation of the deformed covariant derivatives $\mathcal{D}^I$ and $\bar{\mathcal{D}}_J$, we consider instead harmonization of these constraints corresponding to the harmonic space $\mathrm{SU}(4)/[\mathrm{SU}(3)\times\mathrm{U}(1)]$ (E. Ivanov, S. Kalitzin, A.V. Nguyen, V. Ogievetsky, J. Phys. A **18** (1985) 3433) with additional constraints given in the component level. The relevant harmonic superfield $\bar{V}^{(+3)}$ is defined on the analytic harmonic subspace

$$\left\{t_{\mathrm{A}}, \theta^{(+3)}, \bar{\theta}^{(+)\alpha}, u^{(+)\alpha}_I, u^{(+3)I}, u^{(-)I}_\beta, u^{(-3)}_I\right\}.$$

The superfield $\bar{V}^{(+3)}$ satisfies the harmonic constraints

$$\mathcal{D}^{(+4)\alpha}\bar{V}^{(+3)} = 0\,, \qquad \mathcal{D}^{\alpha}_{\beta}\bar{V}^{(+3)} = 0\,, \qquad \mathcal{D}^{0}\bar{V}^{(+3)} = 3\bar{V}^{(+3)}.$$

Here, $\bar{V}^{(+3)}$ is considered as an unconstrained deformed harmonic superfield. The rest of constraints can be given in the component level requiring the field content to be $(\mathbf{8},\mathbf{8},\mathbf{0})$. Skipping details, the deformed transformations are written as

$$\begin{aligned}
\delta z^{I} &= 2\,\epsilon_{K}\chi^{IK}\,e^{3imt/4} + \sqrt{2}\,\bar{\epsilon}^{I}\,\bar{\chi}\,e^{-3imt/4}\,,\\
\delta \bar{z}_{J} &= -\,2\,\bar{\epsilon}^{K}\chi_{JK}\,e^{-3imt/4} - \sqrt{2}\,\epsilon_{J}\chi\,e^{3imt/4},\\
\delta\chi &= \sqrt{2}\,\bar{\epsilon}^{K}\left(i\dot{\bar{z}}_{K} + \frac{3m}{4}\,\bar{z}_{K}\right)e^{-3imt/4},\\
\delta\bar{\chi} &= -\,\sqrt{2}\,\epsilon_{K}\left(i\dot{z}^{K} - \frac{3m}{4}\,z^{K}\right)e^{3imt/4},\\
\delta\chi^{IJ} &= 2\,\bar{\epsilon}^{[I}\left(i\dot{z}^{J]} + \frac{m}{4}\,z^{J]}\right)e^{-3imt/4} - \varepsilon^{IJKL}\epsilon_{K}\left(i\dot{\bar{z}}_{L} - \frac{m}{4}\,\bar{z}_{L}\right)e^{3imt/4},
\end{aligned}$$

where

$$\overline{(z^{I})} = \bar{z}_{I}\,, \qquad \overline{(\chi)} = \bar{\chi}\,, \qquad \overline{(\chi^{IJ})} = \chi_{IJ} = \frac{1}{2}\,\varepsilon_{IJKL}\,\chi^{KL}.$$

SU(2|1) superfields

- Again, we split the given multiplet into SU(2|1) multiplets as $(\mathbf{4},\mathbf{4},\mathbf{0})\oplus(\mathbf{4},\mathbf{4},\mathbf{0})$. The first multiplet is described by the superfield q^{iA} satisfying the SU(2|1) covariant constraints

$$\mathcal{D}^{(k}q^{i)A}=0\,,\quad \bar{\mathcal{D}}^{(k}q^{i)A}=0\,,\qquad \tilde{F}q^{iA}=-\frac{1}{2}\,(\sigma_3)^A_B\,q^{iB}\,,\qquad \overline{(q^{iA})}=q_{iA}\,.$$

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$$\tilde{F}Y^a=\frac{1}{2}\,Y^a,\qquad \tilde{F}\bar{Y}^a=-\,\frac{1}{2}\,\bar{Y}^a,\qquad \overline{(Y^a)}=\bar{Y}_a\,.$$

Invariant action

The general SU(2|1) invariant action is written as

$$S = \int dt\,\mathcal{L} = \frac{1}{2}\int dt\,d^2\theta\,d^2\bar{\theta}\left(1 + 2m\,\bar{\theta}^k\theta_k\right) f\left(Y^a\bar{Y}_a, q^{iA}q_{iA}\right),$$

where the target metric G is defined as

$$\Delta_y = -\,2\,\varepsilon^{ab}\partial_a\bar{\partial}_b\,,\qquad \Delta_x = \varepsilon^{ij}\varepsilon^{AB}\partial_{iA}\partial_{jB}\,,$$
$$G := \Delta_y f = -\,\Delta_x f \quad\Rightarrow\quad (\Delta_y + \Delta_x)\,G = 0\,,$$

Then we require SU(4) invariance of this action that gives the following conditions:

$$m\left(2\partial_a f + \bar{y}_a G + x^{iA}\partial_{iA}\partial_a f\right) = 0 \;\;\Rightarrow\;\; m\left(\bar{y}_a\partial_{iA} - x_{iA}\partial_a\right)G = 0\,, \qquad \text{c.c.}$$

The only solution of these equations is given by

$$f = \frac{1}{4}\left(y^a\bar{y}_a\right)^{-1}\left(y^a\bar{y}_a + \frac{1}{2}\,x^{iA}x_{iA}\right)^{-1} \quad\Rightarrow\quad G = \left(y^a\bar{y}_a + \frac{1}{2}\,x^{iA}x_{iA}\right)^{-3}.$$

The metric is SO(8)-invariant that corresponds to OSp(8|2) superconformal solution. Indeed, this solution gives the same OSp(8|2) superconformal Lagrangian and superconformal transformations are equivalent for both multiplets.

Summary

- We have shown the existence of two non-equivalent “root” multiplets $(\mathbf{8},\mathbf{8},\mathbf{0})$ of the deformed $\mathcal{N}=8$ supersymmetry associated with the supergroup $\mathrm{SU}(4|1)$.

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Thank you for your attention!