On three-dimensional massive higher spin supermultiplets

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TSPU, Tomsk

International conference QFTG-2018
Tomsk, 31 July, 2018

Plan

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It is well known that for higher spins the notion of 3D spin is defined only for massive fields. In massless case there are no degrees of freedom.

Two degrees of freedom of 3D massive higher spin fields can be generated:

- by adding Chern-Simons-like term - Topologically massive HS theory
  S. Deser, R. Jackiw and S. Templeton 1982

  Supersymmetric extantions:

  Off-shell $N = 1, 2$ superfield description for the topologically massive HS
  S. Kuzenko, M. Tsulaia; S. Kuzenko, D. Ogburn 2016

- by adding usual massive term similar to 4D
  I. Tyutin and M. Vasiliev 1997

  Supersymmetric extantions:

  The conditions defying the $N = 1, 2$ massive HS superfield representations
  S. Kuzenko, J. Novak, G. Tartaglino-Mazzucchelli 2015

  $N = 1$ unfolded component equations
  I. Buchbinder, T.S, Yu. Zinoviev 2016

  Goal:

  On-shell $N = 1$ component Lagrangian description
  I. Buchbinder, T.S, Yu. Zinoviev 2017 (arXiv:1711.11450)

Supersymmetry

- In three dimensions N-extended AdS supersymmetry has several incarnations. It is so-called (p, q) supersymmetries where p, q are non-negative integers and $N = p + q$ A. Achucarro, P. Townsend 1986
- The simplest case is $(1, 0)$ supersymmetry. It is naturally associated with 3D AdS supergroup

$$OSp(1, 2) \otimes Sp(2)$$

It means that any massive supermultiplets must contain one bosonic and one fermionic degrees of freedom. For arbitrary interger s we have two higher-spin supermultiplets $(s, s + \frac{1}{2})$ and $(s, s - \frac{1}{2})$.

Frame-like multispinor formalism

- 3D AdS background frame is described by 1-form $e^{\alpha\beta}$ $(\alpha, \beta = 1, 2)$. Double and triple product is defined as

$$\text{two-form} \quad E^{\alpha\beta} = \frac{1}{4} e^{\alpha}{}_{\gamma} e^{\gamma\beta}, \quad \text{three-form} \quad E = \frac{1}{6} E_{\alpha\beta} e^{\alpha\beta}$$

- AdS covariant derivative D is normalized as

$$D^2 \xi^{\alpha} = -\lambda^2 E^{\alpha}{}_{\beta} \zeta^{\beta}$$

- The parameter of global supertransformations ζ^{α} satisfies the relation

$$D\zeta^{\alpha} = -\frac{\lambda}{2} e^{\alpha}{}_{\beta} \zeta^{\beta}$$

- 3D Minkowski case is reproduced by limit $\lambda \to 0$ and $D \to d$.

Massless (gauge) fields
Frame-like formulation

- Spin $k \geq 2$: physical 1-forms $f^{\alpha(2k-2)}$ and axillary 1-forms $\Omega^{\alpha(2k-2)}$

$$\mathcal{L}_k = (-1)^k [(k-1)\Omega_{\alpha(2k-3)\beta} e^\beta{}_\gamma \Omega^{\alpha(2k-3)\gamma} + \Omega_{\alpha(2k-2)} df^{\alpha(2k-2)}]$$

$$\delta f^{\alpha(2k)} = d\xi^{\alpha(2k)} + e^\alpha{}_\beta \eta^{\alpha(2k-1)\beta}, \quad \delta\Omega^{\alpha(2k)} = d\eta^{\alpha(2k)}$$

- Spin 1: physical 1-form A and axillary 0-form $B^{\alpha\alpha}$

$$\mathcal{L}_1 = EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}dA, \qquad \delta A = d\xi$$

- Spin 0: physical 0-form φ and axillary 0-form $\pi^{\alpha\alpha}$

$$\mathcal{L}_0 = -E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}d\varphi$$

Schematically: f_k is physical fields, Ω_k is axillary one

$$\mathcal{L}_k = \Omega_k\Omega_k + \Omega_k df_k, \qquad \delta f_k = d\xi_k + \eta_k, \quad \delta\Omega_k = d\eta_k$$

Massive fields as system of massless ones

I. Buchbinder, T.S, Yu. Zinoviev 2012

In gauge invariant form massive spin s can be described as system of massless one $s \geq k \geq 0$ coupled by the Stueckelberg symmetries.

Fields variables

$$\sum_{k=0}^{s}(f_k, \Omega_k) = \sum_{k=2}^{s}(f^{\alpha(2k-2)}, \Omega^{\alpha(2k-2)}) + (A, B^{\alpha\alpha}) + (\varphi, \pi^{\alpha\alpha})$$

Lagrangian

$$\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{cross} + \mathcal{L}_{mass}$$

$$\begin{aligned} \mathcal{L}_{kinetic} &= \sum_k (\Omega_k \Omega_k + \Omega_k D f_k) \\ \mathcal{L}_{cross} &= m \sum_k (\Omega_k f_{k-1} + f_k \Omega_{k-1}) \\ \mathcal{L}_{mass} &= m^2 \sum_k f_k f_k \end{aligned}$$

Gauge transformations

$$\begin{aligned} \delta f_k &= D\xi_k + \eta_k + m(\xi_{k-1} + \xi_{k+1}) \\ \delta \Omega_k &= D\eta_k + m(\eta_{k-1} + \eta_{k+1}) + m^2 \xi_k \end{aligned}$$

Free higher spin fermion

Massless (gauge) fields
Frame-like formulation

- Spin $k+1/2,\ (k \geq 1)$: physical 1-forms $\Phi^{\alpha(2k-1)}$

$$\mathcal{L}_k = (-1)^k \frac{i}{2} \Phi_{\alpha(2k-1)} d\Phi^{\alpha(2k-1)}, \qquad \delta\Phi^{\alpha(2k-1)} = d\xi^{\alpha(2k-1)}$$

- Spin $1/2$: physical 0-form ϕ^α

$$\mathcal{L}_0 = \frac{i}{2} \phi_\alpha E^\alpha{}_\beta d\phi^\beta$$

Schematically: $\Phi_{k+\frac{1}{2}}$ is physical fields

$$\mathcal{L}_k = \Phi_{k+\frac{1}{2}} d\Phi_{k+\frac{1}{2}}, \qquad \delta\Phi_{k+\frac{1}{2}} = d\xi_{k+\frac{1}{2}}$$

Massive fields as system of massless ones

I. Buchbinder, T.S, Yu. Zinoviev 2014

In gauge invariant form massive spin $s+\frac{1}{2}$ can be described as system of massless one $s+\frac{1}{2}\geq k+\frac{1}{2}\geq 1/2$ coupled by the Stueckelberg symmetries.

Fields variables

$$\sum_{k=0}^{s}\Phi_{k+\frac{1}{2}}=\sum_{k=1}^{s}\Phi^{\alpha(2k-1)}+\phi^{\alpha}$$

Lagrangian

$$\mathcal{L}=\mathcal{L}_{kinetic}+\mathcal{L}_{cross}+\mathcal{L}_{mass}$$

$$\begin{aligned}\mathcal{L}_{kinetic} &= \sum_{k}\Phi_{k+\frac{1}{2}}D\Phi_{k+\frac{1}{2}}\\ \mathcal{L}_{cross} &= m\sum_{k}\Phi_{k+\frac{1}{2}}\Phi_{k-\frac{1}{2}}\\ \mathcal{L}_{mass} &= m\sum_{k}\Phi_{k+\frac{1}{2}}\Phi_{k+\frac{1}{2}}\end{aligned}$$

Gauge transformations

$$\delta\Phi_{k+\frac{1}{2}} = D\xi_{k+\frac{1}{2}}+m(\xi_{k-\frac{1}{2}}+\xi_{k+\frac{3}{2}})+m\xi_{k+\frac{1}{2}}$$

Massless supermultiplet $(k+1, k+\frac{3}{2})$

$$\mathcal{L}_0 = (-1)^{k+1}[k\Omega_{\alpha(2k-1)\beta}e^{\beta}{}_{\gamma}\Omega^{\alpha(2k-1)\gamma} + \Omega_{\alpha(2k)}df^{\alpha(2k)} + \frac{i}{2}\Phi_{\alpha(2k+1)}d\Phi^{\alpha(2k+1)}]$$

Global supertransformations

$$\delta f^{\alpha(2k)} = i(2k+1)\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta, \qquad \delta\Phi^{\alpha(2k+1)} = \alpha_k\Omega^{\alpha(2k)}\zeta^\alpha$$

Massless supermultiplet $(k+1, k+\frac{1}{2})$

$$\mathcal{L}_0 = (-1)^{k+1}[k\Omega_{\alpha(2k-1)\beta}e^{\beta}{}_{\gamma}\Omega^{\alpha(2k-1)\gamma} + \Omega_{\alpha(2k)}df^{\alpha(2k)} - \frac{i}{2}\Phi_{\alpha(2k-1)}d\Phi^{\alpha(2k-1)}]$$

Global supertransformations

$$\delta f^{\alpha(2k)} = i\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha, \qquad \delta\Phi^{\alpha(2k-1)} = 2k\beta_k\Omega^{\alpha(2k-1)\beta}\zeta_\beta$$

Massless higher spin supermultiplets

Massless supermultiplet $(k+\frac{1}{2}, k+1, k+\frac{3}{2})$

$$\mathcal{L}_0 = (-1)^{k+1}[k\Omega_{\alpha(2k-1)\beta}e^{\beta}{}_{\gamma}\Omega^{\alpha(2k-1)\gamma} + \Omega_{\alpha(2k)}df^{\alpha(2k)} - \frac{i}{2}\Phi_{\alpha(2k-1)}d\Phi^{\alpha(2k-1)} + \frac{i}{2}\Phi_{\alpha(2k+1)}d\Phi^{\alpha(2k+1)}]$$

Global supertransformations

$$\delta f^{\alpha(2k)} = i(2k+1)\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta + i\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha$$
$$\delta\Phi^{\alpha(2k-1)} = 2k\beta_k\Omega^{\alpha(2k-1)\beta}\zeta_\beta, \qquad \delta\Phi^{\alpha(2k+1)} = \alpha_k\Omega^{\alpha(2k)}\zeta^\alpha$$

Commutator

$$[\delta_1, \delta_2]f^{\alpha(2l)} = i[(2k+1){\alpha_k}^2 + 2k{\beta_k}^2]\Omega^{\alpha(2l-1)\beta}({\zeta_1}_\beta{\zeta_2}^\alpha - {\zeta_2}_\beta{\zeta_1}^\alpha)$$

corresponds to

$$[Q_\alpha, Q_\beta] \sim P_{\alpha\beta}$$

Main idea is to generalize gauge invariant formulation of massive fields to the case of massive supermultiplets.

Massive fields as system of massless ones

$$s \quad \xrightarrow{m=0} \quad s \oplus (s-1) \oplus ... \oplus 0 = \sum_{k=0}^{s} k$$

$$(s+\frac{1}{2}) \quad \xrightarrow{m=0} \quad (s+\frac{1}{2}) \oplus (s-\frac{1}{2}) \oplus ... \oplus \frac{1}{2} = \sum_{k=0}^{s} (k+\frac{1}{2})$$

Massive supermultiplet as system of massless ones

$$(s, s+\frac{1}{2}) \quad \xrightarrow{m=0} \quad (0, \frac{1}{2}, ..., k-\frac{1}{2}, k, k+\frac{1}{2}, ..., s, s+\frac{1}{2})$$

Massive higher spin supermultiplets

General scheme of construction of massive supermultiplet $(s, s+1/2)$

- We start with massless supermultiplet $(0, \frac{1}{2}, ..., k-\frac{1}{2}, k, k+\frac{1}{2}, ..., s, s+\frac{1}{2})$

$$\mathcal{L} = \sum_k (\Omega_k\Omega_k + \Omega_k df_k + \Phi_{k+\frac{1}{2}} d\Phi_{k+\frac{1}{2}})$$

$$\delta f_k = \Phi_{k-\frac{1}{2}}\zeta + \Phi_{k+1/2}\zeta, \qquad \delta\Phi_{k+\frac{1}{2}} = \Omega_k\zeta + \Omega_{k+1}\zeta$$

- We deform it to massive and AdS $d \to D$ case

$$\begin{aligned}\mathcal{L}_m &= m\sum_k(\Omega_k f_{k-1} + f_k\Omega_{k-1}) + m^2\sum_k f_k f_k \\ &+ m\sum_k \Phi_{k+\frac{1}{2}}\Phi_{k-\frac{1}{2}} + m\sum_k \Phi_{k+\frac{1}{2}}\Phi_{k+\frac{1}{2}}\end{aligned}$$

On the equtaion for axillary fields $\Omega_k = Df_k + m(f_{k-1} + f_{k+1})$

$$\delta_m\Phi_{k+\frac{1}{2}} = m(f_k\zeta + f_{k+1}\zeta)$$

Massive higher spin supermultiplets

Initial supertransformations

$$
\begin{aligned}
\delta f^{\alpha(2k)} &= i\beta_k \Phi^{\alpha(2k-1)}\zeta^{\alpha} + i\alpha_k \Phi^{\alpha(2k)\beta}\zeta_{\beta} \\
\delta A &= i\alpha_0 \Phi^{\alpha}\zeta_{\alpha} + i\beta_0 e_{\alpha\beta}\phi^{\alpha}\zeta^{\beta} \\
\delta \varphi &= -\frac{i\tilde{\alpha}_0}{2}\phi^{\gamma}\zeta_{\gamma} \\
\delta \Phi^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\Omega^{\alpha(2k)}\zeta^{\alpha} + 2(k+1)\beta_{k+1}\Omega^{\alpha(2k+1)\beta}\zeta_{\beta} \\
\delta \Phi^{\alpha} &= 2\beta_1 \Omega^{\alpha\beta}\zeta_{\beta} + \alpha_0 e_{\beta(2)} B^{\beta(2)}\zeta^{\alpha} \\
\delta \phi^{\alpha} &= 4\beta_0 B^{\alpha\beta}\zeta_{\beta} + \tilde{\alpha}_0 \pi^{\alpha\beta}\zeta_{\beta}
\end{aligned}
$$

Deformation to massive theory fix all parameters

$$
\begin{aligned}
\alpha_k &= \sqrt{k(s+k+1)\frac{(M+(k+1)\lambda)}{(M+s\lambda)}}\hat{\alpha}, \quad \alpha_0 = \sqrt{(s+1)\frac{(M+\lambda)}{(M+s\lambda)}}\hat{\alpha} \\
\beta_k &= \sqrt{\frac{(k+1)(s-k)}{2k(2k+1)}\frac{(M-k\lambda)}{(M+s\lambda)}}\hat{\alpha}, \quad \tilde{\alpha}_0 = 4\beta_0 = \sqrt{2s\frac{M}{(M+s\lambda)}}\hat{\alpha}
\end{aligned}
$$

Correction to supertransformations

$$\begin{aligned}\delta\Phi^{\alpha(2k+1)} &= \gamma_k f^{\alpha(2k)}\zeta^{\alpha} + \delta_k f^{\alpha(2k+1)\beta}\zeta_{\beta} \\ \delta\Phi^{\alpha} &= \delta_0 f^{\alpha\beta}\zeta_{\beta} + \gamma_0 A\zeta^{\alpha} + \tilde{\gamma}_0\varphi e^{\alpha}{}_{\beta}\zeta^{\beta} \\ \delta\phi^{\alpha} &= \tilde{\delta}_0\varphi\zeta^{\alpha}\end{aligned}$$

$$\begin{aligned}\gamma_k &= \frac{sM}{2k(k+1)(2k+1)}\alpha_k, \quad \gamma_0 = -2\tilde{\gamma}_0 = sM\alpha_0 \\ \delta_k &= \frac{sM}{(k+2)}\beta_{k+1}, \quad \tilde{\delta}_0 = -(s+1)(M+\lambda)\tilde{\alpha}_0\end{aligned}$$

Summary

- Gauge invariant formualtion of 3D massive higher spin fields can be generalized to 3D massive higher spin supermultiplets.
- In such formulation massive supermultiplets is described as system of massless ones which are mixed with each other. Massive deformation fixes the arbitrariness of mixing
- To construct massive deformations for supertransformations we need add terms without derivatives for fermionic fields only.
- General scheme can be directly generalized to construction of 4D massive higher spin supermultiplets.