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How to construct the NSVZ and NSVZ-like schemes

1. There are no divergent quantum corrections to the superpotential.
2. The β-function of $\mathcal{N}=1$ SYM is related to the anomalous dimensions of the matter superfields by the so called NSVZ β-function,

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1-C_2\alpha/2\pi)},\qquad \text{where}$$

$$\text{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB};\qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$

$$f^{ACD} f^{BCD} \equiv C_2\delta^{AB};\qquad r \equiv \delta_{AA}.$$

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

3. The three-point vertices with two lines of the Faddeev–Popov ghosts and one line of the quantum gauge superfield are finite in all orders.

The NSVZ β-function and calculations in the lowest loops

The NSVZ β-function can be compared with the results of calculations in the lowest orders of the perturbation theory. To make such calculations, a theory should be regularized.

The dimensional regularization breaks the supersymmetry and is not convenient for calculations in supersymmetric theories. That is why supersymmetric theories are mostly regularized by the dimensional reduction. However, the dimensional reduction is not self-consistent.

Using the dimensional reduction and $\overline{\mbox{DR}}$-scheme a β-function of $\mathcal{N}=1$ supersymmetric theories was calculated up to the four-loop approximation:

L.V.Avdeev, O.V.Tarasov, Phys.Lett. **B 112** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett. **B 386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

The result coincides with the NSVZ β-function only in one- and two-loop approximations. In the higher loops it is necessary to make a special tuning of the coupling constant.

The higher covariant derivative regularization is a consistent regularization, which does not break supersymmetry.

A.A.Slavnov, Nucl.Phys., **B 31** (1971) 301; Theor.Math.Phys. **13** (1972) 1064.

In order to regularize a theory by higher derivatives it is necessary to add a term with higher degrees of covariant derivatives. Then divergences remain only in the one-loop approximation. These remaining divergences are regularized by inserting the Pauli–Villars determinants.

A.A.Slavnov, Theor.Math.Phys. **33** (1977) 977.

The higher covariant derivative regularization can be generalized to the $\mathcal{N}=1$ supersymmetric case

V.K.Krivoshchekov, Theor.Math.Phys. **36** (1978) 745;
P.West, Nucl.Phys. **B 268** (1986) 113.

In this talk we will mostly discuss quantum corrections in SUSY theories regularized by higher covariant derivatives.

The NSVZ relation in $\mathcal{N}=1$ SQED

The NSVZ β-function for $\mathcal{N}=1$ SQED with N_f flavours has the form

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big).$$

M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. **42** (1985) 224; Phys.Lett. **B 166** (1986) 334.

This equation is obtained in all loops for the renormalization group functions (RGFs) defined in terms of the bare coupling constant

$$\beta\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) \equiv \frac{d\alpha_0(\alpha,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}};$$

$$\gamma\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) \equiv -\frac{d\ln Z(\alpha,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}}$$

in the case of using the higher derivative regularization

K.S., Nucl.Phys. **B 852** (2011) 71; JHEP **1408** (2014) 096.

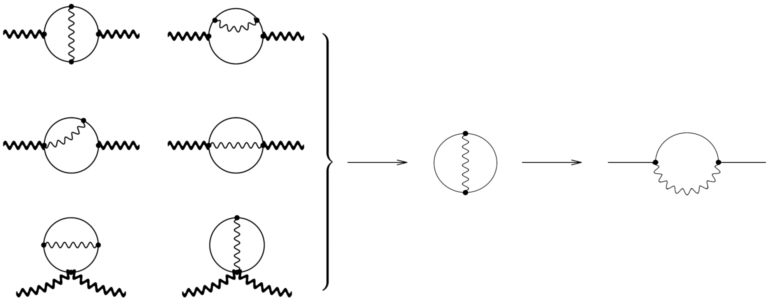
independently of the subtraction scheme, because these RGFs are scheme-independent for a fixed regularization.

The NSVZ β-function for the Abelian case in the supergraph language

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0}$$
$$= \frac{N_f}{\pi}\Big(1 - \frac{d}{d\ln\Lambda}\ln G(\alpha_0,\Lambda/q)\Big|_{q=0}\Big) = \frac{N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big).$$

Qualitative picture for $\mathcal{N}=1$ SQED

A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704** (2005) 445.



The NSVZ scheme for $\mathcal{N}=1$ SQED

RGFs are usually defined in terms of the renormalized coupling constant,

$$\widetilde{\beta}\Big(\alpha(\alpha_0,\Lambda/\mu)\Big) \equiv \frac{d\alpha(\alpha_0,\Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}};$$

$$\widetilde{\gamma}\Big(\alpha(\alpha_0,\Lambda/\mu)\Big) \equiv \frac{d\ln Z(\alpha(\alpha_0,\Lambda/\mu),\Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}}.$$

These RGFs are scheme-dependent. They coincide with RGFs defined in terms of the bare coupling constant, if the boundary conditions

$$Z_3(\alpha,x_0)=1;\qquad Z(\alpha,x_0)=1$$

are imposed on the renormalization constants, where x_0 is an arbitrary fixed value of $\ln\Lambda/\mu$.

A.L.Kataev and K.S., Nucl.Phys. **B 875** (2013) 459; Phys.Lett. **B 730** (2014) 184; Theor.Math.Phys. **181** (2014) 1531.

In this case

$$\frac{\widetilde{\beta}(\alpha)}{\alpha^2}=\frac{N_f}{\pi}\Big(1-\widetilde{\gamma}(\alpha)\Big).$$

The (three-loop) renormalized coupling constant for $\mathcal{N}=1$ SQED can be calculated for the higher derivative regulator $R_k = 1 + k^{2n}/\Lambda^{2n}$:

$$\frac{1}{\alpha_0} = \frac{1}{\alpha} - \frac{N_f}{\pi}\Big(\ln\frac{\Lambda}{\mu} + b_1\Big) - \frac{\alpha N_f}{\pi^2}\Big(\ln\frac{\Lambda}{\mu} + b_2\Big) - \frac{\alpha^2 N_f}{\pi^3}\Big(\frac{N_f}{2}\ln^2\frac{\Lambda}{\mu} - \ln\frac{\Lambda}{\mu}\Big(N_f\sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2} - N_f b_1\Big) + b_3\Big) + O(\alpha^3),$$

where b_i are arbitrary finite constants.
Similarly, the renormalization constant Z (in the two-loop approximation) for the matter superfields is not also uniquely defined:

$$Z = 1 + \frac{\alpha}{\pi}\Big(\ln\frac{\Lambda}{\mu} + g_1\Big) + \frac{\alpha^2(N_f+1)}{2\pi^2}\ln^2\frac{\Lambda}{\mu} - \frac{\alpha^2}{\pi^2}\ln\frac{\Lambda}{\mu}\Big(N_f\sum_{I=1}^{n} c_I \ln a_I - N_f b_1 + N_f + \frac{1}{2} - g_1\Big) + \frac{\alpha^2 g_2}{\pi^2} + O(\alpha^3),$$

where g_i are other arbitrary finite constants.
The subtraction scheme is fixed by values of the constants b_i and g_i.

RGFs defined in terms of the bare coupling constant are

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{N_f}{\pi} + \frac{\alpha_0 N_f}{\pi^2} - \frac{\alpha_0^2 N_f}{\pi^3}\Big(N_f\sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2}\Big) + O(\alpha_0^3);$$

$$\gamma(\alpha_0) = -\frac{\alpha_0}{\pi} + \frac{\alpha_0^2}{\pi^2}\Big(N_f\sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2}\Big) + O(\alpha_0^3).$$

They do not depend on the finite constants b_i and g_i (i.e. they are scheme-independent) and satisfy the NSVZ relation.

RGFs defined in terms of the renormalized coupling constant are

$$\frac{\widetilde{\beta}(\alpha)}{\alpha^2} = \frac{N_f}{\pi} + \frac{\alpha N_f}{\pi^2} - \frac{\alpha^2 N_f}{\pi^3}\Big(N_f\sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2} + N_f(b_2 - b_1)\Big) + O(\alpha^3)$$

$$\widetilde{\gamma}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{\pi^2}\Big(N_f + \frac{1}{2} + N_f\sum_{I=1}^{n} c_I \ln a_I - N_f b_1 + N_f g_1\Big) + O(\alpha^3)$$

and depend on a subtraction scheme.

NSVZ = HD+MSL

The NSVZ scheme is determined by the conditions

$$\alpha_0(\alpha_{\mathrm{NSVZ}}, x_0) = \alpha_{\mathrm{NSVZ}}; \qquad Z_{\mathrm{NSVZ}}(\alpha_{\mathrm{NSVZ}}, x_0) = 1$$

For $x_0 = 0$ these conditions give

$$g_1 = g_2 = b_1 = b_2 = b_3 = \ldots = 0.$$

Therefore, only powers of $\ln \Lambda/\mu$ are included into the renormalization constants. This is similar to minimal subtrations, so that this scheme can be called HD+MSL, i.e. Higher Derivative regularization + Minimal Subtraction of Logarithms. In this case in the considered approximations

$$\frac{\widetilde\beta(\alpha)}{\alpha^2} = \frac{N_f}{\pi} + \frac{\alpha N_f}{\pi^2} - \frac{\alpha^2 N_f}{\pi^3}\Big(N_f \sum_{I=1}^{n} c_I \ln a_I + N_f + \frac{1}{2}\Big) + O(\alpha^3) = \frac{\beta(\alpha)}{\alpha^2};$$

$$\widetilde\gamma(\alpha) = \frac{d\ln Z}{d\ln\mu} = -\frac{\alpha}{\pi} + \frac{\alpha^2}{\pi^2}\Big(N_f + \frac{1}{2} + N_f \sum_{I=1}^{n} c_I \ln a_I\Big) + O(\alpha^3) = \gamma(\alpha).$$

Thus, we see that in this scheme the NSVZ relation is really satisfied.

Let us consider $\mathcal{N}=1$ SQCD interacting with the Abelian gauge field. In this case the gauge group is $G\times U(1)$. In the massless limit this theory is described by the action

$$S=\frac{1}{2g_0^2}\,\mbox{tr}\,\mbox{Re}\int d^4x\,d^2\theta\,W^a W_a+\frac{1}{4e_0^2}\mbox{Re}\int d^4x\,d^2\theta\,\boldsymbol{W}^a\boldsymbol{W}_a$$
$$+\sum_{f=1}^{N_f}\frac{1}{4}\int d^4x\,d^4\theta\Big(\phi_f^+ e^{2q_f\boldsymbol{V}+2V}\phi_f+\widetilde\phi_f^+ e^{-2q_f\boldsymbol{V}-2V^t}\widetilde\phi_f\Big),$$

where V is a non-Abelian gauge superfield (corresponding to the non-Abelian group G), and $\boldsymbol{V}$ is the Abelian gauge superfield (corresponding to the group $U(1)$). The considered theory contains two coupling constants:

$$\alpha_s=\frac{g^2}{4\pi}\qquad\mbox{and}\qquad\alpha=\frac{e^2}{4\pi}.$$

The Adler D-function

S. L. Adler, Phys. Rev. D **10** (1974) 3714.

encodes quantum corrections to the electromagnetic coupling contant α, which appear due to the quark loop with internal gluon and quark lines. The diagrams containing internal photon lines are omitted. (Thus, the electromagnetic field $\boldsymbol{V}$ is treated as an external field.)
Due to the Ward identity the two-point Green function of the superfield $\boldsymbol{V}$ is transversal,

$$\Delta\Gamma^{(2)} = -\frac{1}{16\pi}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \boldsymbol{V}\, \partial^2\Pi_{1/2}\boldsymbol{V}\, \Big(d^{-1}(\alpha_0,\alpha_{0s},\Lambda/p) - \alpha_0^{-1}\Big).$$

We will consider the Adler function defined in terms of the bare coupling constant by the equation

$$D(\alpha_{0s}) = \frac{3\pi}{2}\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\alpha_{0s},\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = \frac{3\pi}{2\alpha_0^2}\frac{d\alpha_0}{d\ln\Lambda}.$$

Exact expression for the Adler function

It is possible to derive the NSVZ-like exact expression for the Adler function in the considered theory.

M.A.Shifman and K.S., Phys.Rev.Lett. **114** (2015) 051601; Phys.Rev. **D 91** (2015) 105008.

In the case of an arbitrary gauge group and an arbitrary representation for the matter superfields it has the form

$$D(\alpha_{s0}) = \frac{3}{2}\sum_{\alpha=1}^{N_f} q_\alpha^2 \Big(\text{dim}(R) - \text{tr}\,\gamma(\alpha_{s0})\Big).$$

It is valid in all loops in the case of using the higher derivative regularization. For RGFs defined in terms of the renormalized couplings the NSVZ-like relation is also valid in all loops with the HD+MSL prescription

A. L. Kataev, A. E. Kazantsev and K.S., Nucl. Phys. B **926** (2018) 295.

$$Z(\alpha_s, x_0)_i{}^j = \delta_i{}^j; \qquad Z_\alpha(\alpha,\alpha_s,x_0) = 1; \qquad Z_{\alpha_s}(\alpha_s,x_0) = 1,$$

so that $$\widetilde{D}(\alpha_s) = \frac{3}{2}\sum_{\alpha=1}^{N_f} q_\alpha^2 \Big(\text{dim}(R) - \text{tr}\,\widetilde{\gamma}(\alpha_s)\Big).$$

Exact renormalization of the photino mass in the softly broken $\mathcal{N}=1$ SQED

The integrals defining the anomalous dimension of the photino mass

$$\gamma_m(\alpha_0) \equiv \frac{d\ln m_0}{d\ln\Lambda}$$

in softly broken $\mathcal{N}=1$ SQED regularized by higher derivatives are also integrals of double total derivatives in all loops.

I.V.Nartsev, K.S., JHEP **1704** (2017) 047; JETP Lett. **105** (2017) 69.

This can be proved by the generalization of the method described above and leads to the NSVZ-like relation

$$\gamma_m(\alpha_0) = \frac{\alpha_0 N_f}{\pi}\Big[1 - \frac{d}{d\alpha_0}\Big(\alpha_0\gamma(\alpha_0)\Big)\Big].$$

J.Hisano, M.A.Shifman, Phys.Rev. **D56** (1997) 5475;
I.Jack, D.R.T.Jones, Phys.Lett. **B415** (1997) 383;
L.V.Avdeev, D.I.Kazakov, I.N.Kondrashuk, Nucl.Phys. **B510** (1998) 289.

The NSVZ-like scheme (for RGFs defined in terms of the renormalized coupling constant) in this case is defined by the HD+MSL conditions

$$Z_3(\alpha, x_0) = 1; \qquad Z(\alpha, x_0) = 1; \qquad Z_m(\alpha, x_0) = 1.$$

Let us consider the (massless) non-Abelian $\mathcal{N}=1$ SYM theory with matter

$$S = \frac{1}{2e_0^2}\,\mbox{Re}\,\mbox{tr}\int d^4x\, d^2\theta\, W^a W_a + \frac{1}{4}\int d^4x\, d^4\theta\, \phi^{*i}(e^{2\mathcal{F}(V)})_i{}^j\phi_j$$
$$+\Big\{\frac{1}{6}\lambda_0^{ijk}\int d^4x\, d^2\theta\, \phi_i\phi_j\phi_k + \mbox{c.c.}\Big\},$$

where the matter superfields ϕ_i belong to a representation R of the gauge group, and Yukawa couplings λ_0 satisfy the condition

$$\lambda_0^{ijm}(T^A)_m{}^k + \lambda_0^{imk}(T^A)_m{}^j + \lambda_0^{mjk}(T^A)_m{}^i = 0.$$

Note that usually $\mathcal{F}(V) = V$, but for calculating quantum corrections we should use this function, see below. Then the supersymmetric gauge superfield strength is given by

$$W_a \equiv \frac{1}{8}\bar D^2\Big(e^{-2\mathcal{F}(V)}D_a e^{2\mathcal{F}(V)}\Big).$$

Quantum-background splitting is made by the substitution

$$e^{2\mathcal{F}(V)} \to e^{\boldsymbol{\Omega}^+} e^{2\mathcal{F}(V)} e^{\boldsymbol{\Omega}}.$$

The background superfield $\boldsymbol{V}$ is defined by $e^{2\boldsymbol{V}} = e^{\boldsymbol{\Omega}^+} e^{\boldsymbol{\Omega}}$.
We choose the higher derivative term

$$S_\Lambda = \frac{1}{2e_0^2}\,\mbox{Re}\,\mbox{tr}\int d^4x\, d^2\theta\, e^{\boldsymbol{\Omega}} W^a e^{-\boldsymbol{\Omega}} \Big[R\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big) - 1\Big]_{Adj} e^{\boldsymbol{\Omega}} W_a e^{-\boldsymbol{\Omega}}$$
$$+\frac{1}{4}\int d^4x\, d^4\theta\, \phi^+ e^{\boldsymbol{\Omega}^+} e^{2\mathcal{F}(V)} \Big[F\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big) - 1\Big] e^{\boldsymbol{\Omega}}\phi$$

and the gauge fixing term

$$S_{\mbox{gf}} = -\frac{1}{16\xi_0 e_0^2}\mbox{tr}\int d^4x\, d^4\theta\, \boldsymbol{\nabla}^2 V K\Big(-\frac{\bar{\boldsymbol{\nabla}}^2\boldsymbol{\nabla}^2}{16\Lambda^2}\Big)_{Adj} \bar{\boldsymbol{\nabla}}^2 V,$$

where the regulators R, F, and K have a rapid growth at infinity.

Actions for the Faddeev–Popov and Nielsen–Kallosh ghosts have the form

$$S_{\text{FP}} = \frac{1}{2}\int d^4x\, d^4\theta\, \frac{\partial \mathcal{F}^{-1}(\widetilde{V})^A}{\partial \widetilde{V}^B}\,\Big|_{\widetilde{V}=\mathcal{F}(V)} \Big(e^{\mathbf{\Omega}}\bar{c}e^{-\mathbf{\Omega}} + e^{-\mathbf{\Omega}^+}\bar{c}^+ e^{\mathbf{\Omega}^+}\Big)^A$$

$$\times \bigg\{\Big(\frac{\mathcal{F}(V)}{1-e^{2\mathcal{F}(V)}}\Big)_{Adj}\Big(e^{-\mathbf{\Omega}^+}c^+e^{\mathbf{\Omega}^+}\Big) + \Big(\frac{\mathcal{F}(V)}{1-e^{-2\mathcal{F}(V)}}\Big)_{Adj}\Big(e^{\mathbf{\Omega}}ce^{-\mathbf{\Omega}}\Big)\bigg\}^B$$

$$S_{\text{NK}} = \frac{1}{2e_0^2}\text{tr}\int d^4x\, d^4\theta\, b^+\Big[e^{\mathbf{\Omega}^+} K\Big(-\frac{\bar{\mathbf{\nabla}}^2\mathbf{\nabla}^2}{16\Lambda^2}\Big)e^{\mathbf{\Omega}}\Big]_{Adj} b.$$

The total action of the gauge fixed theory is invariant under the BRST transformations and the background gauge transformations.
Below we will see that the quantum gauge superfield V is renormalized nonlinearly. Parameters describing this nonlinear renormalization are included into the function $\mathcal{F}(V)$. Calculating quantum corrections in the lowest loops in the Feynman gauge it is possible to set $\mathcal{F}(V) = V$. However, we will demonstrate that the nonlinear renormalization of the quantum gauge superfield is very essential.

In our notation the renormalization constants are defined by the equations

$$\frac{1}{\alpha_0} = \frac{Z_\alpha}{\alpha}; \qquad \frac{1}{\xi_0} = \frac{Z_\xi}{\xi}; \qquad \boldsymbol{V} = \boldsymbol{V}_R; \qquad \bar{c}c = Z_c Z_\alpha^{-1} \bar{c}_R c_R;$$

$$b = \sqrt{Z_b}\, b_R; \qquad V = Z_V Z_\alpha^{-1/2} V_R + \text{nonlinear terms};$$

$$\phi_i = (\sqrt{Z_\phi})_i{}^j (\phi_R)_j; \qquad \lambda^{ijk} = \lambda_0^{mnp} (Z_\lambda)_m{}^i (Z_\lambda)_n{}^j (Z_\lambda)_p{}^k.$$

The subscript R denotes renormalized superfields, α, λ, and ξ are the renormalized coupling constant, the Yukawa couplings, and the gauge parameter, respectively.

It is possible to impose the following constrains to these renormalization constants:

$$(Z_\lambda)_i{}^j = (\sqrt{Z_\phi})_i{}^j; \qquad Z_\xi = Z_V^{-2}; \qquad Z_b = Z_\alpha^{-1}.$$

Non-renormalization of the triple gauge-ghost vertices

The three-point vertices with two ghost legs and a single leg of the quantum gauge superfield are finite in all orders

K.S., Nucl.Phys. **B909** (2016) 316.

There are 4 such vertices, $\bar{c}Vc$, $\bar{c}^+Vc$, $\bar{c}Vc^+$, and $\bar{c}^+Vc^+$.
They have the same renormalization constant $Z_\alpha^{-1/2}Z_cZ_V$. Therefore, the above statement can be rewritten as

$$\frac{d}{d\ln\Lambda}(Z_\alpha^{-1/2}Z_cZ_V) = 0.$$

Earlier, similar equations have been derived in the Landau gauge $\xi\to 0$ for the usual Yang–Mills theory

D. Dudal, H. Verschelde, S.P. Sorella, Phys. Lett. B555 (2003) 126

and for $\mathcal{N}=1$ SYM in the Wess–Zumino gauge (also in the case $\xi\to 0$)

M.A.L. Capri, D.R. Granado, M.S. Guimaraes, I.F. Justo, L. Mihaila, S.P. Sorella, D. Vercauteren, Eur.Phys.J. C74 (2014) no.4, 2844.

The Green functions of the structure $\bar{c}V^nc$ are divergent for $n\ne 1$.

Renormalization group functions defined in terms of the bare couplings

We will define RGFs in terms of the bare couplings by the equations

$$\beta(\alpha_0,\lambda_0) \equiv \frac{d\alpha_0}{d\ln\Lambda};$$

$$(\gamma_\phi)_i{}^j(\alpha_0,\lambda_0) \equiv -\frac{d\ln(Z_\phi)_i{}^j(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda};$$

$$\gamma_V(\alpha_0,\lambda_0) \equiv -\frac{d\ln Z_V(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda};$$

$$\gamma_c(\alpha_0,\lambda_0) \equiv -\frac{d\ln Z_c(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda},$$

where the differentiation is made at fixed values of α and λ^{ijk}.
These renormalization group functions are

1. scheme independent at a fixed regularization;
2. depend on a regularization;
2. satisfy the NSVZ relation in all orders for $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives.

The NSVZ β-function can be equivalently rewritten in the form

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{3C_2 - T(R) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r}{2\pi} + \frac{C_2}{2\pi}\cdot\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0}.$$

Let us express the β-function in the right hand side in terms of the renormalization constant Z_α:

$$\beta(\alpha_0,\lambda_0) = \frac{d\alpha_0(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\alpha_0\frac{d\ln Z_\alpha}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}}.$$

Then, using the identity $d(Z_\alpha^{-1/2}Z_V Z_c)/d\ln\Lambda = 0$ we obtain

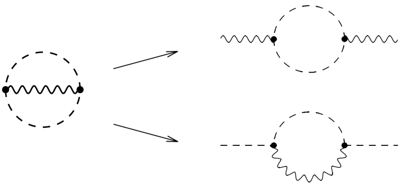
$$\beta(\alpha_0,\lambda_0) = -2\alpha_0\frac{d\ln(Z_c Z_V)}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = 2\alpha_0\Big(\gamma_c(\alpha_0,\lambda_0)+\gamma_V(\alpha_0,\lambda_0)\Big),$$

where γ_c and γ_V are anomalous dimensions of the Faddeev–Popov ghosts and of the quantum gauge superfield (defined in terms of the bare coupling constants), respectively.

Substituting this expression into the right hand side of the NSVZ relation we obtain

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\gamma_c(\alpha_0,\lambda_0)$$
$$-2C_2\gamma_V(\alpha_0,\lambda_0) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r\Big).$$

From this form of the NSVZ β-function we see that the matter superfields and ghosts similarly contribute to the right hand side.



The graphical interpretation is similar to the Abelian case

Renormalization group functions defined in terms of the renormalized couplings

RGFs are defined in terms of the renormalized couplings by the equations

$$\widetilde{\beta}(\alpha,\lambda) \equiv \frac{d\alpha}{d\ln\mu};$$

$$(\widetilde{\gamma}_\phi)_i{}^j(\alpha,\lambda) \equiv \frac{d\ln(Z_\phi)_i{}^j(\alpha_0,\lambda_0,\Lambda/\mu)}{d\ln\mu};$$

$$\widetilde{\gamma}_V(\alpha,\lambda) \equiv \frac{d\ln Z_V(\alpha_0,\lambda_0,\Lambda/\mu)}{d\ln\mu};$$

$$\widetilde{\gamma}_c(\alpha,\lambda) \equiv \frac{d\ln Z_c(\alpha_0,\lambda_0,\Lambda/\mu)}{d\ln\mu},$$

where the differentiation is made at fixed values of α_0 and λ_0^{ijk}.
These renormalization group functions are
1. scheme and regularization dependent;
2. satisfy the NSVZ relation only for a special renormalization prescription, called the NSVZ scheme.

The NSVZ scheme in the non-Abelian case

RGFs defined in terms of the renormalized coupling constant are scheme dependent and satisfy the NSVZ relation only in a certain subtraction scheme. Similarly to

A.L.Kataev and K.S., Nucl.Phys. **B875** (2013) 459; Phys.Lett. **B730** (2014) 184.

we see that in the non-Abelian case RGFs defined in terms of the bare coupling constant coincide with ones defined in terms of the renormalized coupling constants if the boundary conditions

$$Z_\alpha(\alpha,\lambda,x_0) = 1; \qquad (Z_\phi)_i{}^j(\alpha,\lambda,x_0) = \delta_i{}^j; \qquad Z_c(\alpha,\lambda,x_0) = 1,$$

where x_0 is a fixed value of $\ln\Lambda/\mu$, are imposed on the renormalization constants. We also assume that $Z_V = Z_\alpha^{1/2} Z_c^{-1}$.
For $x_0 = 0$ only powers of $\ln\Lambda/\mu$ are included into the renormalization constants, so that we obtain the HD+MSL prescription. Possibly,

HD+MSL=NSVZ

This has been verified in the three-loop approximation for terms containing the Yukawa couplings.

NSVZ relation and the Faddeev–Popov ghosts

However, the terms containing the ghost anomalous dimension γ_c in the equation

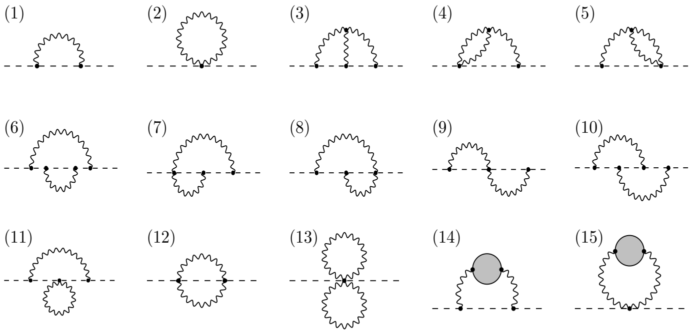
$$\frac{\widetilde{\beta}(\alpha,\lambda)}{\alpha^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\widetilde{\gamma}_c(\alpha,\lambda) - 2C_2\widetilde{\gamma}_V(\alpha,\lambda) + C(R)_i{}^j\left(\widetilde{\gamma}_\phi\right)_j{}^i(\alpha,\lambda)/r\Big)$$

have been verified only for the two-loop β-function and the one-loop anomalous dimensions

V.Yu.Shakhmanov, K.S., Phys.Lett. B **776** (2018) 417.

However, a nontrivial check can be obtain only by comparing the two-loop ghost anomalous dimension and the three-loop β-function, because only stating from this approximation the scheme dependence becomes essential. That is here we describe the calculation of the two-loop anomalous dimension of the Faddeev–Popov ghosts.

The two-loop anomalous dimension of the Faddeev–Popov ghosts is contributed to by the superdiagrams



where the gray circle denotes insertion of the one-loop polarization operator calculated in

A.E.Kazantsev, M.B.Skoptsov, K.S., Mod.Phys.Lett. A **32** (2017) no.36, 1750194.

The ghost anomalous dimension can be obtained from the function G_c which is constructed according to the prescription

$$\Gamma_c^{(2)} = \frac{1}{4}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta \Big(c^{*A}(-p,\theta)\bar{c}^A(p,\theta) + \bar{c}^{*A}(-p,\theta)c^A(p,\theta)\Big)$$
$$\times G_c(\alpha_0,\lambda_0,\xi_0,y_0,\ldots,\Lambda/p).$$

In terms of bare couplings the ghost anomalous dimension is defined by

$$\gamma_c(\alpha_0,\lambda_0,\xi_0,y_0) \equiv -\frac{d\ln Z_c}{d\ln\Lambda}\Big|_{\alpha,\lambda,\xi,y=\text{const}} = \frac{d\ln G_c}{d\ln\Lambda}\Big|_{\alpha,\lambda,\xi,y=\text{const};\, p=0}.$$

Note that for calculating this expression in the considered approximation we have to take into account nonlinear terms inside the function $\mathcal{F}(V)$,

$$\mathcal{F}^A(V) = V^A + e_0^2\, y_0\, G^{ABCD} V^B V^C V^D + \ldots, \qquad \text{where}$$

$$G^{ABCD} = \frac{1}{6}\Big(f^{AKL} f^{BLM} f^{CMN} f^{DNK} + \text{permutations of } B,\ C, \text{ and } D\Big)$$

The nonlinear renormalization of the quantum gauge superfield V was first discussed in

O.Piguet and K.Sibold, Nucl.Phys. B **196** (1982) 428; B **197** (1982) 257; 272; B **248** (1984) 301; I.V.Tyutin, Yad.Fiz. **37** (1983) 761.

Explicit calculations (for gauge superfield four-point Green function)

J.W.Juer and D.Storey, Phys.Lett. **119B** (1982) 125; Nucl.Phys. B **216** (1983) 185.

demonstrated that the nonlinear terms really appear. In our notation this implies that

$$y_0 = y + \frac{\alpha}{90\pi}\Big((2+3\xi)\ln\frac{\Lambda}{\mu} + k_1\Big) + \ldots$$

Also we need the one-loop renormalization of some other parameters:

$$\alpha_0 = \alpha - \frac{\alpha^2}{2\pi}\Big[3C_2\Big(\ln\frac{\Lambda}{\mu} + b_{11}\Big) - T(R)\Big(\ln\frac{\Lambda}{\mu} + b_{12}\Big)\Big] + O(\alpha^3, \alpha^2\lambda^2);$$

$$\alpha_0\xi_0 = \alpha\xi + \frac{\alpha^2 C_2}{3\pi}\Big(\xi(\xi-1)\ln\frac{\Lambda}{\mu} + x_1\Big) + O(\alpha^3, \alpha^2\lambda^2).$$

The result for the ghost anomalous dimension defined in terms of the bare coupling constant has been obtained in

A.E. Kazantsev, M.D. Kuzmichev, N.P. Meshcheriakov, S.V. Novgorodtsev, I.E. Shirokov, M.B. Skoptsov, K.S., JHEP 1806 (2018) 020.

for the higher derivative regulators $R(x) = K(x) = 1+x^m$; $F(x) = 1+x^n$,

$$\gamma_c(\alpha_0,\lambda_0,\xi_0,y_0) = \frac{\alpha_0 C_2(\xi_0-1)}{6\pi} - \frac{5\alpha_0 y_0 C_2^2(\xi_0-1)}{4\pi} - \frac{\alpha_0^2 C_2^2}{24\pi^2}\Big(\xi_0^2-1\Big) - \frac{\alpha_0^2 C_2^2}{4\pi^2}\Big(\ln a_\varphi + 1\Big) + \frac{\alpha_0^2 C_2 T(R)}{12\pi^2}\Big(\ln a + 1\Big) + \ldots,$$

where $a \equiv M/\Lambda$; $a_\varphi \equiv M_\varphi/\Lambda$. Note that we keep the one-loop y-dependence, but omit the dependence on the nonlinearity parameters in the two-loop terms. (In the two-loop approximation it is necessary to take into account other parameters describing the nonlinear renormalization.) In agreement with the general arguments the result is scheme-independent.

Two-loop ghost anomalous dimension defined in terms of the renormalized couplings

In terms of the renormalized couplings the ghost anomalous dimension is defined as

$$\widetilde{\gamma}_c(\alpha,\lambda,\xi,y) = \frac{d\ln Z_c}{d\ln\mu}\bigg|_{\alpha_0,\lambda_0,\xi_0,y_0=\text{const}}.$$

The result is (h_1 is finite constant inside Z_c)

$$\widetilde{\gamma}_c(\alpha,\lambda,\xi,y) = \frac{\alpha C_2(\xi-1)}{6\pi} - \frac{5\alpha y C_2^2(\xi-1)}{4\pi} - \frac{\alpha^2 C_2^2}{4\pi^2}\Big(\ln a_\varphi + 1 + 6h_1 - b_{11}\Big) + \frac{\alpha^2 C_2 T(R)}{12\pi^2}\Big(\ln a + 1 + 6h_1 - b_{12}\Big) - \frac{\alpha^2 C_2^2}{24\pi^2}\Big(\xi^2-1\Big) + \frac{\alpha^2 C_2^2}{72\pi^2} \times\Big(4x_1 - (\xi-1)k_1\Big) + \ldots$$

We see that the result is scheme dependent. Also it can be easily verified that in the HD+MSL scheme (for which $b_{11}=b_{12}=0$, $h_1=0$, $k_1=0$, $x_1=0$) it coincides with γ_c after a formal substitution

$$\alpha\to\alpha_0;\qquad \xi\to\xi_0;\qquad y\to y_0.$$

Conclusion

- ✓ In the Abelian case the NSVZ scheme (for RGFs defined in terms of the renormalized couplings) can be constructed in all loops by the help of the HD+MSL prescription, i.e. for theories regularized by Higher Covariant Derivatives and the renormalization prescription by the Minimal Subtraction of Logarithms.
- ✓ The HD+MSL prescription also gives the NSVZ-like schemes for the Alder D-function in $\mathcal{N}=1$ SQCD and for the renormalization of the photino mass in softly broken $\mathcal{N}=1$ SQED.
- ✓ In the non-Abelian case the NSVZ scheme seems to be also produced by HD+MSL, but the perturbative calculations lead to the relation between the β-function and the anomalous dimensions of the quantum superfields. The usual NSVZ relation is obtained by the help of the non-renormalization theorem for the triple gauge-ghost vertices.
- ✓ Renormalization of supersymmetric theories in higher orders requires the nonlinear renormalization of the quantum gauge superfield.

Thank you for the attention!