Cosmology in modified $f(R, T)$-gravity.

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Action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} F(R,\ T) + \epsilon \int d^4x \sqrt{-g} L_m,$$

$$\Downarrow$$

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} F(R,\ T, \Box T) + \epsilon \int d^4x \sqrt{-g} L_m,$$

Stability conditions

$$S = \int d^4x\sqrt{-g}F(R,\ T, \Box T) =$$

$$\int d^4x\sqrt{-g}[F(\lambda_1, \lambda_2, \lambda_3) + \mu_1(R - \lambda_1) + \mu_2(T - \lambda_2) + \mu_3(\Box T - \lambda_3)],$$

variation with respect to μ_i

$$\lambda_1 = R,\ \ \lambda_2 = T,\ \ \lambda_3 = \Box T,$$

and variation with respect to λ_i:

$$\mu_1 = F_{\lambda_1},\ \ \mu_2 = F_{\lambda_2},\ \ \mu_3 = F_{\lambda_3},$$

$$S = \int d^4x\sqrt{-g}[\mu_1 R + \mu_3\Box\lambda_2 + \{F(\lambda_1, \lambda_2, \lambda_3) - \mu_1\lambda_1 - \mu_3\lambda_3\}].$$

new fields $\lambda_2 = \chi_2 + \psi_2$ and $\mu_3 = \chi_2 - \psi_2$

$$S = \int d^4x\sqrt{-g}\mu_3\Box\lambda_2 =$$

$$\int d^4x\sqrt{-g}[\chi_2\Box\chi_2 - \psi_2\Box\chi_2 + \chi_2\Box\psi_2 - \psi_2\Box\psi_2] =$$

$$\int d^4x\sqrt{-g}[-\nabla^i\chi_2\nabla_i\chi_2 + \nabla^i\psi_2\nabla_i\psi_2]$$

one special case

$$F(R,\ T, \Box T) = f(R,\ T) + h(T)\Box T.$$

introducing auxiliary fields as $\lambda = R,\ \ \mu = f_\lambda$

$$S = \int d^4x\sqrt{-g}[\mu R + V(\mu, \lambda, T) - h' g^{ik}\nabla_i T\nabla_k T],$$

producing conformal transformation of the metric $\bar{g}_{ik} = e^\chi g_{ik}$,
$\chi = \ln \mu$ we have the action in canonical form

$$S{=}\int d^4x\sqrt{-\bar{g}}[\bar{R}{-}\frac{3}{2}\bar{g}^{ik}\bar{\nabla}_i\chi\bar{\nabla}_k\chi{+}e^{-2\chi}V(\mu, \lambda, T){-}h' e^{-\chi}\bar{g}^{ik}\bar{\nabla}_i T\bar{\nabla}_k T].$$

we need $f_R > 0$ and $h' > 0$ for ghost-free theory

$$h'\Box T + \Box h + f_T(R, T) = 2h'\Box T + h''(\nabla^i T)(\nabla_i T) + f_T(R, T) = 0,$$

Under the flat background we find additional restriction $f_{TT} \geqslant 0$ for the absence of tachyon-like effective particles in the theory (case $f_{TT} = 0$ can not be totaly excluded). For more complicate cases of non-flat background this relation will has more complicate structure and will contain h'' also.

Equations of motion

$$F(R,\ T, \Box T) = f(R,\ T) + h(T)\Box T,$$

$$h'\Box T + \Box h + f_T(R, T) = 2h'\Box T + h''(\nabla^i T)(\nabla_i T) + f_T(R, T) = 0,$$

$$f_R R_{ik} - \frac{1}{2}F g_{ik} + (g_{ik}\Box - \nabla_i \nabla_k) f_R = 8\pi\epsilon T_{ik}$$

$$-(f_T + h'\Box T + \Box h)(T_{ik} + \Theta_{ik}) + h'\nabla_i T \nabla_k T - \frac{1}{2}h'\nabla_m T \nabla^m T g_{ik},$$

$$\Theta_{ik} \equiv g^{lm}\frac{\delta T_{lm}}{\delta g^{ik}}$$

$$f_R R_{ik} - \frac{1}{2} F g_{ik} + (g_{ik}\Box - \nabla_i \nabla_k) f_R = 8\pi\epsilon T_{ik} + h' \nabla_i T \nabla_k T - \frac{1}{2} h' \nabla_m T \nabla^m T g_{ik},$$

There is no true limit for $h = 0$ due to field equation.

Conservation equation

divergence of r.h.s.

$$\nabla^i \left[f_R R_{ik} - \frac{1}{2} F g_{ik} + (g_{ik}\Box - \nabla_i \nabla_k) f_R \right] = -\frac{1}{2} f_T \nabla_k T - \frac{1}{2} \nabla_k (h \Box T),$$

where we used

$$\nabla^i G_{ik} = 0, \;\text{ and }\; (\nabla^i f_R) R_{ik} = (\Box \nabla_k - \nabla_k \Box) f_R,$$

$$8\pi\epsilon \nabla^i T_{ik} = -\frac{1}{2} f_T \nabla_k T - \frac{1}{2} \nabla_k (h \Box T) - \nabla^i \left[h' \nabla_i T \nabla_k T \right] + \frac{1}{2} \nabla_k (h' \nabla_l T \nabla^l T),$$

Examples for cosmological applications

$$T_{ik} = (\rho + p)u_i u_k - pg_{ik},$$

where $u_i u^k = 1$ and $u^i \nabla_k u_i = 0$,

$$\Theta_{ik} = -2T_{ik} - pg_{ik}.$$

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),$$

$$p = 0, \quad T = \rho$$

$$f(R,T) = R + 2f(T), \;\; h(T) = \alpha T$$

$$3\frac{\dot{a}^2}{a^2} = 3H^2 = 8\pi\epsilon\rho + f(\rho) + \frac{1}{2}\alpha\dot{\rho}^2 + \frac{1}{2}\alpha\rho(\ddot{\rho} + 3H\dot{\rho}),$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = 2\dot{H} + 3H^2 = f(\rho) - \frac{1}{2}\alpha\dot{\rho}^2 + \frac{1}{2}\alpha\rho(\ddot{\rho} + 3H\dot{\rho}),$$

$$\alpha(\ddot{\rho} + 3H\dot{\rho}) = -f'(\rho),$$

$$8\pi\epsilon(\dot{\rho} + 3H\rho) = \frac{1}{2}f'\dot{\rho} + \frac{1}{2}f''\rho\dot{\rho},$$

$$3H^2\left[1-\frac{\alpha}{2}\frac{3(8\pi\epsilon\rho)^2}{(\frac{1}{2}f'(\rho)+\frac{1}{2}f''\rho-8\pi\epsilon)^2}\right]=8\pi\epsilon\rho-\frac{1}{2}\rho f'+f,$$

which for instance for $f = 2\lambda\rho$ reads

$$3H^2\left[1-\frac{3\alpha}{2}\frac{(8\pi\rho)^2}{(\lambda-8\pi)^2}\right]=(8\pi+\lambda)\rho,$$

and for $f = \lambda\rho^2$, $\epsilon = 1$

$$3H^2\left[1-\frac{3\alpha}{2}\frac{(8\pi\rho)^2}{(2\lambda\rho-8\pi)^2}\right]=8\pi\rho.$$

$$w_{eff} \equiv -1 - 2\dot{H}/H^2,$$

$$w_{eff} = \frac{-2f + \rho f' + \alpha\dot{\rho}^2}{16\pi\rho\epsilon + 2f - \rho f' + \alpha\dot{\rho}^2},$$

$$16\pi\rho \ll 2f - \rho f',$$

$$\dot{\rho}^2 \ll 2f - \rho f',$$

$$w_{eff} = -1,$$

comparison with the limit case $h = 0$

$$3H^2 = 8\pi\epsilon\rho + f + 2\rho f',$$

$$2\dot{H} + 3H^2 = f,$$

which corresponds to eos

$$w_{eff} = \frac{-f}{8\pi\epsilon\rho + f + 2\rho f'},$$

$$f(\rho) = \frac{a_1 \rho^n + b_1 \rho^m}{a_2 \rho^n + b_2 \rho^m},$$

where constants are positive and $n > m > 0$

$$\lim_{\rho \to +\infty} f(\rho) = \frac{a_1}{a_2}, \quad \lim_{\rho \to +0} f(\rho) = \frac{b_1}{b_2},$$

$$3\frac{\dot{a}^2}{a^2} = 3H^2 = 8\pi\epsilon\rho + f(\rho) + \frac{1}{2}\alpha\dot{\rho}^2 + \frac{1}{2}\alpha\rho(\ddot{\rho} + 3H\dot{\rho}),$$

$$\frac{a_1}{a_2} \approx \Lambda_{inf}, \quad \frac{b_1}{b_2} \approx \Lambda_0,$$

Conclusion

- Introducing new term we significantly change initial theory
- Modified theory has more reach and complicate dynamic
- We find some restrictions for the theory and a number of interesting cosmological solutions