On massive super(bi)gravity in the constructive approach

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Outlook

1. Constructive approach
2. Frame-like gauge invariant formalism
3. Cubic vertices
   - Massive spin-2 and massive spin-3/2
   - Massive spin-2 and massive and massless spin-3/2
   - Massive spin-2 and two massive spin-3/2

Construction of the cubic vertices

- We follow the so-called constructive approach

$$\mathcal{L} = \mathcal{L}_0 + g\mathcal{L}_1 + \dots, \qquad \delta\Phi = \delta_0\Phi + g\delta_1\Phi + \mathcal{O}(g^2)$$

  where $\mathcal{L}_0$ is a free (quadratic) Lagrangian, $\mathcal{L}_1$ contains cubic terms and so on, while δ_0 — initial (non-homogeneous) gauge transformations, δ_1 — linear in fields and so on.

- For the cubic level in the frame-like formalism one has to solve

$$\frac{\delta\mathcal{L}_1}{\delta\Phi}\delta_0\Phi + \frac{\delta\mathcal{L}_1}{\delta\Omega}\delta_0\Omega + \frac{\delta\mathcal{L}_0}{\delta\Phi}\delta_1\Phi + \frac{\delta\mathcal{L}_0}{\delta\Omega}\delta_1\Omega = 0$$

- We use a modified 1 and 1/2 order formalism:

$$\left[\frac{\delta\mathcal{L}_1}{\delta\Phi}\delta_0\Phi + \frac{\delta\mathcal{L}_1}{\delta\Omega}\delta_0\Omega + \frac{\delta\mathcal{L}_0}{\delta\Phi}\delta_1\Phi\right]_{\frac{\delta\mathcal{L}_0}{\delta\Omega}=0} = 0$$

Notations

- All fields are one-forms (gauge fields) or zero-forms (Stueckelberg fields) with some set of the local indices
- Coordinate-free description of $d = 4$ Minkowski space uses background frame e^a, its inverse $\hat{e}_a$ and covariant derivative D such that
$$D \wedge D = 0, \qquad D \wedge e^a = 0$$
- We use the following notations for the products of frames:
$$E^{ab} = e^a \wedge e^b, \qquad E^{abc} = e^a \wedge e^b \wedge e^c, \qquad E^{abcd} = e^a \wedge e^b \wedge e^c \wedge e^d$$
and similarly for $\hat{e}_a$
- We use Majorana representation for γ-matrices and define
$$\Gamma^{ab} = \frac{1}{2}\gamma^{[a}\gamma^{b]}, \qquad \Gamma^{abc} = \frac{1}{3!}\gamma^{[a}\gamma^{b}\gamma^{c]}, \qquad \Gamma^{abcd} = \frac{1}{4!}\gamma^{[a}\gamma^{b}\gamma^{c}\gamma^{d]}$$

Massive spin-2

- Frame-like gauge invariant formalism is given by the Lagrangian

$$\begin{aligned}
\mathcal{L}_0 &= \frac{1}{2}\hat{E}_{ab}\Omega^a{}_c\Omega^{bc} - \frac{1}{2}\hat{E}_{abc}\Omega^{ab}Df^c + \frac{1}{2}B_{ab}B^{ab} \\
&\quad -\hat{E}_{ab}B^{ab}DA - \frac{1}{3}\pi_a\pi^a + \frac{2}{3}\hat{e}_a\pi^a D\sigma \\
&\quad +m\hat{E}_{ab}\Omega^{ab}A + m\hat{e}_aB^{ab}f_b - 2m\hat{e}_a\pi^a A \\
&\quad +\frac{m^2}{2}\hat{E}_{ab}f^af^b - m^2\hat{e}_af^a\sigma + \frac{2m^2}{3}\sigma^2
\end{aligned}$$

- which is invariant under the following gauge transformations

$$\begin{aligned}
\delta\Omega^{ab} &= D\eta^{ab} - \frac{m^2}{2}e^{[a}\xi^{b]} \\
\delta f^a &= D\xi^a - e_b\eta^{ab} + me^a\xi \\
\delta B^{ab} &= -m\eta^{ab}, \qquad \delta A = D\xi + \frac{m}{2}e_a\xi^a \\
\delta\pi^a &= -\frac{3m^2}{2}\xi^a, \qquad \delta\sigma = 3m\xi
\end{aligned}$$

Massive spin-3/2

- Frame-like gauge invariant description requires one-form Φ (gravitino) and zero-form ϕ (goldstiino)
- The Lagrangian has the form

$$\begin{aligned} \mathcal{L}_0 &= -\frac{i}{2}\hat{E}_{abc}\bar{\Phi}\Gamma^{abc}D\Phi + \frac{i}{2}\hat{e}_a\bar{\phi}\gamma^a D\phi \\ &\quad -\frac{3m_1}{2}\hat{E}_{ab}\bar{\Phi}\Gamma^{ab}\Phi + 3im_1\hat{e}_a\bar{\Phi}\gamma^a\phi - m_1\bar{\phi}\phi \end{aligned}$$

- and is invariant under the following gauge transformations

$$\delta\Phi = D\zeta + \frac{im_1}{2}e_a\gamma^a\zeta, \qquad \delta\phi = 3m_1\zeta$$

Minimal cubic vertices

- Massive dRGT gravity can be considered as a smooth deformation of the usual massless gravity

$$\mathcal{L} = \mathcal{L}_0(g_{\mu\nu}) - V(g_{\mu\nu}, \eta_{\mu\nu})$$

- An interesting question — are there models that can be considered as smooth deformations of usual (spontaneously broken) supergravity as well as their bigravity versions?
- The first step is to consider cubic vertices for massive spin-2 and massive and/or massless spin-3/2 having no more than one derivative.

Cubic vertex

- The cubic vertex exists and has only one independent coupling constant c_1
- Terms without derivatives

$$\Delta\mathcal{L}_1 = 2m_1 c_1 \hat{E}_{abc}\bar{\Phi}\Gamma^{ab}\Phi f^c + \dots$$

- The limit $m_1 \to 0$ is singular while the limit $m \to 0$ is smooth
- Corrections

$$\delta_1 f^a = -4ic_1\bar{\Phi}\gamma^a\zeta, \qquad \delta_1 A = -\frac{imc_1}{3m_1}\bar{\phi}e_a\gamma^a\zeta, \qquad \delta_1\sigma = -\frac{2c_1}{3}\bar{\phi}\zeta$$

$$\delta_1\Phi = -\frac{c_1}{3}[\Gamma^{ab}\Omega^{ab}\zeta + 2im_1\gamma^a f^a\zeta + 2mc_1 A\zeta + \frac{2im_1}{3}e^a\gamma^a\zeta\sigma]$$

$$\delta_1\phi = \frac{c_1}{3}[\frac{m}{m_1}\Gamma^{ab}B^{ab}\zeta + \frac{4i}{3}\gamma^a\pi^a\zeta - \frac{2(m^2 - 2m_1{}^2)}{m_1}\zeta\sigma]$$

Algebraic structure

- All our fields are either gauge or Stueckelberg ones so even at this order we may consider commutators of bosonic and/or supertransformations
- For the bosonic fields

$$[\delta_1, \delta_2] f^a = D\tilde{\xi}^a - \tilde{\eta}^{ab} e_b, \qquad [\delta_1, \delta_2] A = \frac{m}{2} e_a \tilde{\xi}^a, \qquad [\delta_1, \delta_2]\sigma = 0$$

$$\tilde{\xi}^a = 4ic_1(\bar{\zeta}_2 \gamma^a \zeta_1), \qquad \tilde{\eta}^{ab} = 4m_1 c_1(\bar{\zeta}_2 \Gamma^{ab} \zeta_1), \qquad \tilde{\xi} = 0$$

- For the fermionic fields

$$[\delta_B, \delta_\zeta]\Phi = (D + \frac{im_1}{2} e_a \gamma^a)\tilde{\zeta}, \qquad [\delta_B, \delta_\zeta]\phi = 3m_1 \tilde{\zeta}$$

$$\tilde{\zeta} = -\frac{c_1}{3}(\Gamma^{ab}\eta^{ab}\zeta) - \frac{2im_1 c_1}{3}(\gamma^a \xi^a \zeta) - \frac{2mc_1}{3}(\xi\zeta)$$

Cubic vertex

- Such vertex also exists but for the equal masses

$$m = m_1$$

- It also has one independent coupling constant
- Lagrangian, corrections and algebraic structure can be obtained from the general case with three independent masses m, m_1, m_2 in the limit where $m_2 \to 0$ and $m_1 \to m$

Cubic vertex

- Vertex exists for arbitrary non-zero values for the three masses m, m_1, m_2 with quite complicated dependencies on these masses
- There are two non-singular massless limits
  - the limit $m_2 \to 0$ that requires $m_1 \to m$
  - the limit $m \to 0$ requires $m_1 \to m_2$
- Corrections for the supertransformations have the same general pattern: terms coming from the initial massless supermultiplets plus low derivative terms

Algebraic structure

- Commutators of two supertransformations on the bosonic fields produce

$$
\begin{aligned}
[\delta_1, \delta_2] f^a &= D\tilde{\xi}^a - \tilde{\eta}^{ab} e_b + m e^a \tilde{\xi} \\
[\delta_1, \delta_2] A &= D\tilde{\xi} + \frac{m}{2} e_a \tilde{\xi}^a, \quad [\delta_1, \delta_2]\sigma = 3m\tilde{\xi}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\xi}^a &= 6ic_1(\bar{\zeta}_2 \gamma^a \zeta_1), \quad \tilde{\eta}^{ab} = 3(m_1 + m_2)c_1(\bar{\zeta}_2 \Gamma^{ab} \zeta_1) \\
\tilde{\xi} &= -\frac{3(m_1 - m_2)c_1}{m}(\bar{\zeta}_2 \zeta_1)
\end{aligned}
$$

- while commutators of the bosonic and supertransformations for the first fermion give

$$
[\delta_B, \delta_\zeta]\Phi = (D + \frac{im_1}{2} e_a \gamma^a)\tilde{\zeta}_1, \qquad [\delta_B, \delta_\zeta]\phi = 3m_1 \tilde{\zeta}_1
$$

$$
\tilde{\zeta}_1 = -\frac{c_1}{2}(\Gamma^{ab}\eta^{ab}\zeta_2) - im_1 c_1(\gamma^a \xi^a \zeta_2) - \frac{(m^2 - 2{m_1}^2 + 2{m_2}^2)c_1}{m}(\xi \zeta_2)
$$

Conclusion

- For the spontaneously broken supergravity the cubic vertex for massless spin-2 f^a and massive spin-3/2 (Φ, ϕ) is universal and does not depend on the presence of any other fields in the system.
- Beyond cubic order there are two main possibilities
  - non-linear realisation a la Volkov-Akulov
  - linear realisation where each field is a member of some supermultiplet
- The same holds for the massive supergravities. For example the vertex for massive spin-2 and massive and massless spin-3/2 must corresponds to bigravity:

$$\begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \oplus \begin{pmatrix} & 2 & \\ \frac{3}{2} & & \frac{3}{2} \\ & 1 & \end{pmatrix}$$